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The real business cycle (RBC) approach to the study of aggregate fluctuations is now a well-established paradigm in macroeconomics. Most RBC models abstract from government fiscal policy altogether or treat it as some exogenous stochastic process. This article develops an RBC model in which government fiscal variables such as tax rates and public expenditures are *endogenous*. The authors characterize the "optimal" behavior of fiscal policy over the business cycle for two different tax structures and relate this behavior to movements in private-sector variables like output, consumption, labor hours, and investment. As a benchmark, they also provide a comparison between the model and U.S. data.

Cross-Lender Variation in Home Mortgage Lending 15

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This study evaluates the feasibility of using information collected under the Home Mortgage Disclosure Act (HMDA) to form quantitative measures of lender activity for use in enforcement. By evaluating three firm-level measures—loan application, approval, and origination rates—the authors find that cross-lender variation in minority and low-income originations primarily reflects differences in home mortgage application rates, not in approval rates. The authors also compare gross measures of lender performance with indices controlling for property location and loan applicant characteristics and determine that they perform similarly. This suggests that most of the variation in lender behavior is idiosyncratic and cannot be attributed to variance in applicant characteristics reported in the HMDA data or to differences in the geographic markets served by the lenders.

The Efficiency and Welfare Effects of Tax Reform: Are Fewer Tax Brackets Better than More? 30

by David Altig and Charles T. Carlstrom

On the wish list of many members of the new Congress is an income tax system characterized by constant marginal tax rates, typically referred to as a flat-tax system. In reality, what we are likely to see is a continuation of the worldwide trend toward replacing systems with high marginal-rate progressivity with those that have a smaller number of rates that are flat over relatively broad income ranges. In this article, the authors compare a simple two-bracket tax code with an approximation to traditional structures that entail steeply rising marginal tax rates. Their conclusion — that the simpler rate structures are not necessarily more efficient than alternatives with numerous, highly progressive brackets — serves as a cautionary note to potential reformers.

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Tax Structure, Optimal Fiscal Policy, and the Business Cycle

by Jang-Ting Guo and Kevin J. Lansing

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Introduction

The real business cycle (RBC) approach to the study of aggregate fluctuations is now a well-established paradigm in macroeconomics. The early groundbreaking articles in this area (Kydland and Prescott [1980a, 1982] and Long and Plosser [1983]) completely abstracted from government behavior, yet were reasonably successful in capturing the broad comovements and relative variabilities of the economic aggregates that characterize the business cycle.

More recently, researchers have introduced elements of government fiscal policy into these models to help match various business cycle facts. For example, Christiano and Eichenbaum (1992) include stochastic government spending in the household utility function to help explain the low observed correlation between labor hours and real wages (as measured by average labor productivity) in postwar U.S. data. This works in their model because shocks to government spending impact the marginal utility of private consumption and thereby induce shifts in the household labor supply. These interact with labor demand shifts (caused by technology shocks) to produce a low correlation between wages and hours. Braun (1994) and

McGrattan (1994) show that a similar result can be obtained by introducing stochastic distortionary taxes to shift the labor supply curve. A common feature of these studies is that government policy is viewed as exogenous.

In this paper, we develop an RBC model in which government fiscal variables such as tax rates and public expenditures are *endogenous*. Our objective is to characterize the “optimal” behavior of these policy variables over the business cycle and to relate this behavior to movements in private-sector variables like output, consumption, labor hours, and investment. As a benchmark, we also provide a comparison between the model and U.S. data.

We build on the recent work of Chari, Christiano, and Kehoe (1994), who develop a competitive RBC model in which a government policymaker chooses an optimal sequence of distortionary taxes on labor and capital income in a dynamic version of the Ramsey (1927) optimal tax problem. Our model differs from theirs in three main respects. First, we introduce monopoly profits into the production sector of the economy such that the optimal steady-state tax on capital is positive, consistent with U.S. observations. In a competitive model, this tax rate is zero (see Judd [1985] and Chamley [1986]). Second, our model

incorporates the “indivisible labor” specification of Rogerson (1988) and Hansen (1985). In standard RBC models (which abstract from government), the indivisible labor specification serves to increase the variability of hours relative to the real wage to a value that is more in line with U.S. data. Third, we endogenize the time series of government spending by including in household preferences a separable term that represents the utility provided by public goods. In the Chari, Christiano, and Kehoe model, government spending follows an exogenous stochastic process.

We compare simulations from our model to post-WWII, annual U.S. data and an otherwise similar model with nondistortionary lump-sum taxes. Our results can be summarized as follows: For a given stochastic process of the technology shock, we find that optimal distortionary taxes reduce the variability of output and labor hours, but increase the relative variability of household investment, compared to the model with lump-sum taxes. This result can be traced to the behavior of the optimal distortionary tax rates on labor and capital income. The optimal labor tax in the model is procyclical, which reduces the variability of hours (and output) by providing households with an implicit insurance mechanism against variations in their after-tax wage. The optimal capital tax in the model is countercyclical and displays a high standard deviation relative to the labor tax. This tends to increase the variability of household investment relative to output, but provides an efficient means of absorbing shocks to the government’s budget (which are caused by changes in the size of the tax base over the business cycle).

In addition, we find that the distortionary tax model underpredicts the variability of hours worked relative to the real wage in U.S. data, despite our specification of indivisible labor. This result is due to the procyclical optimal labor tax, which tends to reduce the variability of hours worked in comparison to a standard RBC model with indivisible labor. Finally, both model versions capture the procyclical behavior of government spending in annual U.S. data, but underpredict its variability over the business cycle. We find that this comparison, as well as comparisons along some other dimensions, are substantially improved if we exclude U.S. data prior to 1954 to avoid the influence of the Korean War. However, a few comparisons, such as the correlation between government spending and output, become worse.

The remainder of the paper is organized as follows: Sections I and II describe the model and the solution method. The choice of parameter

values is discussed in section III. Section IV examines the business cycle characteristics of the two tax structures and compares them to U.S. data. Concluding remarks are presented in section V.

I. The Model

The model economy consists of three types of agents: households, firms, and the government. Households obtain direct utility from government-provided public goods, which are financed by taxes on households and firms. Following Benhabib and Farmer (1994), we postulate that firms which produce intermediate goods exhibit some degree of monopoly power such that they realize positive economic profits even though the final-goods sector of the economy is perfectly competitive. The profits are equal to the difference between the value of output and the payments made to inputs. The reason for introducing profits is to obtain a positive optimal tax rate on capital under the distortionary tax structure, consistent with U.S. observations.¹

As owners of the firms, households receive net profits in the form of dividends. It is assumed that profits are initially taxed at the firm level, then distributed as dividends and taxed again at the household level. This formulation is intended to capture the double taxation of corporate dividends in the U.S. economy. Furthermore, under the distortionary tax structure, we assume that the government can distinguish between labor and capital income, but cannot distinguish between the various categories of capital income, such as profits, dividends, bond interest, and capital rental income. Therefore, this version of the model includes only two types of distortionary taxes: a labor tax and a capital tax.

■ 1 Jones, Manuelli, and Rossi (1993) show that the existence of profits and a restriction on the menu of available tax instruments (the absence of a separate profits tax) is one method of obtaining a positive optimal tax rate on capital in the steady state. Without profits, the optimal steady-state tax on capital is zero.

The Household's Problem

There is a continuum of identical, infinitely lived households, each of which maximizes a stream of discounted utilities over sequences of consumption and leisure:

$$(1) \quad \max_{c_t, b_t, k_{t+1}, b_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - A b_t + B \ln g_t)$$

$$0 < \beta < 1, \quad A, B > 0.$$

In this utility function, β is the household discount factor and c_t represents private consumption goods. The symbol E_t is the expectation operator conditional on information available at time t . Each household is endowed with one unit of time each period and works b_t hours during period t . The fact that utility is linear in hours worked draws on the formulation of *indivisible labor* described by Rogerson (1988) and Hansen (1985). This means that all fluctuations in labor hours are due to changes in the number of workers employed, as opposed to variations in hours per worker.² Household preferences also include a term representing the utility provided by aggregate public consumption goods g_t . The separability in c_t and g_t implies that public consumption does not affect the marginal utility of private consumption, a specification supported by parameter estimates in McGrattan, Rogerson, and Wright (1993). Households view g_t as outside their control. Examples of public consumption goods that might affect household utility are national defense, police protection, and government provision of food and shelter during natural disasters. Public goods are assumed to be noncongestable and free of specific user charges.

The representative household faces the following within-period budget constraint:

$$(2) \quad c_t + x_t + b_{t+1} \leq (1 - \tau_{bt}) w_t b_t + (1 - \tau_{kt}) (r_t k_t + \hat{\pi}_t + r_{bt} b_t) + \tau_{kt} \delta k_t + b_t - T_t,$$

$$k_0, b_0 \text{ given,}$$

where x_t is investment, k_t is the stock of physical capital, and b_{t+1} represents one-period, real government bonds carried into period $t+1$ by the household. Households derive income by supplying labor and capital services to firms at rental rates w_t and r_t , and pay taxes on labor and capital income at rates τ_{bt} and τ_{kt} , respectively.

Two additional sources of household income are

the firm's net profits, $\hat{\pi}_t$ (which are distributed to households as dividends), and the interest earned on government bonds, $r_{bt} b_t$. Dividends and interest are taxed at the same rate as capital rental income, $r_t k_t$. The term $\tau_{kt} \delta k_t$ represents the depreciation allowance built into the U.S. tax code, and T_t is a lump-sum tax.

The following equation describes the law of motion for the capital stock, given a constant rate of depreciation δ :

$$(3) \quad k_{t+1} = (1 - \delta) k_t + x_t, \quad 0 < \delta < 1.$$

Households view tax rates, wages, interest rates, and dividends as determined outside their control.

Household Optimality

The household first-order conditions with respect to the indicated variables and the associated transversality conditions (TVC) are

$$(4a) \quad c_t: \lambda_t = \frac{1}{c_t}$$

$$(4b) \quad b_t: \lambda_t (1 - \tau_{bt}) w_t = A$$

$$(4c) \quad k_{t+1}: \lambda_t = \beta E_t \lambda_{t+1} [(1 - \tau_{kt+1}) (r_{t+1} - \delta) + 1]$$

$$(4d) \quad b_{t+1}: \lambda_t = \beta E_t \lambda_{t+1} [(1 - \tau_{kt+1}) r_{bt+1} + 1]$$

$$(4e) \quad \text{TVC: } \lim_{t \rightarrow \infty} E_0 \beta^t \lambda_t k_{t+1} = 0, \quad \lim_{t \rightarrow \infty} E_0 \beta^t \lambda_t b_{t+1} = 0,$$

where λ_t is the Lagrange multiplier associated with the budget constraint (2) in period t . The interpretation of λ_t is that it represents the

■ 2 The linearity of (1) in b_t implies that the effective labor-supply elasticity of the representative household is infinite. In a decentralized economy, both Rogerson and Hansen show that this utility function can be supported by a lottery that randomly assigns workers to employment or unemployment each period, with the firm providing full unemployment insurance. Wage contracts call for households to be paid based on their expected (rather than actual) number of hours worked. RBC models with indivisible labor are better able to match some key characteristics of aggregate labor market data. Specifically, U.S. data display a large variability of hours worked relative to the real wage, and a weak correlation between hours and the real wage (see Christiano and Eichenbaum [1992]).

marginal utility of an additional unit of after-tax income received in period t . The transversality conditions ensure that the household's within-period budget constraint (2) can be transformed into an infinite-horizon, present-value budget constraint.

The Firm's Problem

This section closely follows the model developed by Benhabib and Farmer (1994). Suppose there exists a continuum of intermediate goods y_{it} , $i \in [0,1]$ and a unique final good y_t that is produced using the following constant-returns-to-scale technology:

$$(5) \quad y_t = \left[\int_0^1 y_{it}^\chi di \right]^{1/\chi}, \quad 0 < \chi \leq 1.$$

We assume that the final-goods sector is perfectly competitive, but that intermediate-goods producers exert a degree of monopoly power that is captured by the parameter χ . In the special case when $\chi = 1$, all intermediate goods are perfect substitutes in the production of the final good, and the intermediate sector becomes perfectly competitive.

Each intermediate good is produced using the *same* technology, with labor and capital as inputs:

$$(6) \quad y_{it} = \exp(z_t) k_{it}^{\alpha_1} h_{it}^{\alpha_2},$$

$$0 < \alpha_1 < 1, \alpha_1 + \alpha_2 = 1$$

$$(7) \quad z_{t+1} = \rho_z z_t + \varepsilon_{t+1}, \quad 0 < \rho_z < 1,$$

$$\varepsilon_t \sim \text{i.i.d. } (0, \sigma_\varepsilon^2), \quad z_0 \text{ given,}$$

where (7) is the law of motion for aggregate technology shocks z_t , which are revealed to agents at the beginning of period t and which generate business cycle fluctuations in the model. Under the assumptions that firms maximize profits and factor markets are competitive, Benhabib and Farmer show that in a *symmetric* equilibrium ($k_{it} = k_t$ and $h_{it} = h_t$ for all i), the aggregate production function, the rental rate on capital, and the real wage are

$$(8a) \quad y_t = \exp(z_t) k_t^{\alpha_1} h_t^{\alpha_2},$$

$$(8b) \quad r_t = \theta_1 \frac{y_t}{k_t}, \quad \theta_1 \equiv \chi \alpha_1, \text{ and}$$

$$(8c) \quad w_t = \theta_2 \frac{y_t}{h_t}, \quad \theta_2 \equiv \chi \alpha_2.$$

Due to their monopoly power, intermediate-goods producers earn an economic profit that is taxed at rate τ_{kt} . The firm's after-tax profits, distributed to households in the form of dividends, are

$$(9a) \quad \hat{\pi}_t = (1 - \tau_{kt}) (y_t - r_t k_t - w_t h_t).$$

$$(9b) \quad \hat{\pi}_t = (1 - \tau_{kt}) (1 - \theta_1 - \theta_2) y_t.$$

The Government's Problem

The government chooses an optimal program of taxes, borrowing, and public expenditures in order to maximize the discounted utility of the household. This is a dynamic version of the Ramsey (1927) optimal tax problem, where τ_{bt} , τ_{kt} , r_{bt} , g_t , and T_t summarize government policy implemented at time t . To set up this problem, we begin by spelling out some important assumptions. First, we assume that the government can *commit* to a set of time-invariant decision rules that specify policy variables as a function of state variables. This is done to avoid the complicating issue of *time inconsistency*, which arises in policy design problems when the tax base includes fixed assets (such as capital or bonds) that cannot be quickly adjusted in response to a change in the level of the tax. In these situations, the government has an incentive to deviate from its originally announced, optimal policy by implementing surprise increases in asset taxes in order to obtain nondistortionary tax revenue. Because households understand that the original policy is time inconsistent, it cannot be supported as an equilibrium unless the government can commit itself (and all successor governments) to carrying out the plan.³

Second, given that the initial stocks of capital and bonds are fixed, we rule out any confiscatory taxes on assets at $t=0$ that might be used to finance all future expenditures. This case is not very interesting because no taxes beyond the initial period are required. With these assumptions, the government's problem is

■ 3 The time inconsistency problem does not arise under the lump-sum tax structure because taxes are nondistortionary. See Chari (1994) for a summary of the issues and a review of the literature dealing with time inconsistency problems and optimal policy design.

$$(10) \max_{\substack{c_t, \tau_{bt}, \tau_{kt} \\ r_{bt}, w_t}} E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t - A b_t + B \ln g_t)$$

subject to

- (i) household first-order conditions and the budget constraint,
- (ii) firm profit-maximization conditions,
- (iii) $g_t = b_{t+1} - b_t(1 + r_{bt})$
 $+ \tau_{bt} w_t b_t + \tau_{kt} [(r_t - \delta) k_t + r_{bt} b_t]$
 $+ [1 - (1 - \tau_{kt})^2] (1 - \theta_1 - \theta_2) y_t + T_t,$
- (iv) $T_t = 0$ for the distortionary tax structure,
- (v) $\tau_{bt} = \tau_{kt} = 0$ for the lump-sum tax structure,
- (vi) $\lim_{t \rightarrow \infty} \frac{b_t}{\prod_{i=1}^t [1 + (1 - \tau_{kt-1}) r_{bt-1}]} = 0.$

As a condition for equilibrium, government policy must take into account the rational responses of households and firms, as summarized by constraints (i) and (ii). The chosen policy must also satisfy the government's within-period budget constraint (iii), where the squared term on the right-hand side reflects the double taxation of firm dividends. Constraints (iv) and (v) impose the restrictions associated with the two tax structures we intend to analyze. For the distortionary tax structure, we rule out the possibility of lump-sum taxes. For the lump-sum tax structure, we set τ_{bt} and τ_{kt} equal to zero. Finally, (vi) is a transversality condition ensuring that the government budget constraint is satisfied in present-value terms. The summation of the household budget constraint and the government budget constraint yields the following resource constraint for the economy:

$$(11) \quad y_t = c_t + x_t + g_t.$$

Because the resource constraint and the government budget constraint are not independent equations, equation (11) will be used in place of (iii) in solving the government's problem.

II. Solving the Model

Our approach to solving the government's problem is to find the allocations c_t , b_t , k_{t+1} , and b_{t+1} that maximize household utility subject to the constraints, where allocations are expressed as functions of the economy's state variables. The appropriate set of prices r_t and w_t and the policy variables τ_{bt} , τ_{kt} , r_{bt} , g_t , and T_t that decentralize the optimal allocations can be computed using the profit-maximization conditions (8), the household first-order conditions (4), the household budget constraint (2), and the resource constraint (11).⁴ For example, the optimal allocations uniquely determine λ_t and w_t through equations (4a) and (8c). Given λ_t and w_t , the household's first-order condition for b_t , equation (4b), uniquely determines the government's optimal choice for τ_{bt} . The government has much more flexibility, however, in choosing the optimal capital tax and the optimal interest rate on government debt. The expectation operators in the household's first-order conditions for k_{t+1} and b_{t+1} , equations (4c) and (4d), imply that the after-tax returns on capital and bonds (weighted by marginal utility) must be the same "on average." In response to a series of shocks, the government can satisfy this ex ante arbitrage condition and implement the optimal allocations using many different combinations for the period-by-period values of τ_{kt} and r_{bt} . Consequently, the stochastic version of the model does not uniquely pin down the time-series behavior of these policy variables (see Zhu [1992] and Chari, Christiano, and Kehoe [1994] for a more complete description).

To facilitate a comparison with U.S. data, we make a particular assumption about the way in which the government picks τ_{kt} and r_{bt} to decentralize a set of allocations. Specifically, we employ the certainty versions of (4c) and (4d) to identify τ_{kt} and r_{bt} each period. Requiring the government to satisfy the certainty versions of these constraints guarantees that the uncertainty versions will also be satisfied. Essentially, we are restricting the policy instruments available to the government by ruling out fully flexible, state-contingent capital taxes and bond interest rates. This might be interpreted as reflecting the political infeasibility of some types of policy regimes. The restriction we impose has an impact on the behavior of the allocations in response to stochastic shocks, as does any other restriction

■ 4 This method of solving the government's problem is described as the "primal" approach by Atkinson and Stiglitz (1980), chapter 12.

on the set of available policy instruments (such as ruling out lump-sum taxes). Consequently, the allocations we compute in response to shocks are different from the “Ramsey allocations” that could be supported in an unrestricted environment.⁵ In the restricted case, the government’s decision rules for τ_{kt} and r_{bt} are identical to those for an economy with no uncertainty. It is important to note that this result follows from a particular decentralization scheme. However, our solution method also employs a linear-quadratic approximation of the problem. Thus, the decision rules governing household allocations also display the property of certainty equivalence.⁶

Given these assumptions, the government’s problem with distortionary taxes can be solved using a recursive algorithm developed by Kydland and Prescott (1980b). The problem with lump-sum taxes can be solved by adopting the view of a social planner for an appropriately defined “pseudo-economy” in which the planner cannot exploit the monopoly power of firms. The government’s problem under lump-sum taxes is not equivalent to a standard social planning problem because when $\chi < 1$, the decentralized equilibrium is not Pareto optimal. The pseudo-economy approach is an indirect method of obtaining the equilibrium allocations (see Stokey and Lucas [1989], chapter 18). Government debt does not appear in the pseudo-planner’s problem. This reflects the well-known “Ricardian proposition,” which states that government debt policy is irrelevant to the determination of equilibrium allocations in an economy with lump-sum taxes (see Sargent [1987], chapter 3). Since debt doesn’t matter in this case, we arbitrarily set it equal to zero each period such that $g_t = T_t$. The pseudo-planner’s problem is recursive and can be solved using standard methods.⁷

III. Calibrating the Model to the U.S. Economy

To explore the quantitative predictions of the model, we assign parameter values based on empirically observed features of post-WWII U.S. data. The time period in the model is taken to be one year, which is consistent with both the time frame of most government fiscal decisions and the frequency of available data on average marginal tax rates. The discount factor β ($= 0.962$) implies an annual rate of time preference of 4 percent. The parameter A in the household utility function is chosen such

that the fraction of time spent working is close to 0.3 in the steady state for each tax structure. This coincides with time-use studies, such as Juster and Stafford (1991), which indicate that households spend approximately one-third of their discretionary time in market work. The value of B is chosen to yield a steady-state value of g/y near 0.22 for each tax structure, the average ratio of government spending to GNP for the U.S. economy from 1947 to 1992.⁸ The steady-state level of government debt is chosen to yield a steady-state ratio of b/y equal to 0.45. This is the average value of U.S. federal debt held by the public as a fraction of GNP from 1947 to 1992.⁹

The exponents in the Cobb–Douglas production function are chosen on the basis of two criteria. First, the selected values of θ_1 ($= 0.31$) and θ_2 ($= 0.60$) are in the range of the estimated shares of GNP received by capital and labor in the U.S. economy (see Christiano [1988]). Second, the model’s share of output devoted to monopoly profits ($= 1 - \theta_1 - \theta_2$) is chosen to yield a reasonable value for the steady-state tax on capital (τ_k) under the distortionary tax structure. Because a separate profits tax is not available in this case, the government uses the tax on private capital to recapture a portion of the profits. In the model, the steady-state ratio of profits to output is 0.09, and the

■ 5 See Chari, Christiano, and Kehoe (1994) for examples of decentralizations that support the Ramsey allocations. See Cassou (forthcoming) for a case where policy instruments are restricted to follow a univariate Markov process in response to government spending shocks.

■ 6 The approximate version of the problem involves the maximization of a quadratic objective function subject to linear constraints. Since the first-order conditions are linear in all variables, the expectation operator can be passed through the expressions, dropping out stochastic terms associated with the technology shock innovation ϵ_{t+1} in equation (7). See Sargent (1987), p. 36.

■ 7 A technical appendix to this paper, available from the authors upon request, describes the details of our solution procedure.

■ 8 The specific parameter values used in the computations are $A = 2.50$, $B = 0.350$ for the distortionary tax structure, and $A = 3.48$, $B = 0.381$ for the lump-sum tax structure.

■ 9 The model does not pin down a unique value for the steady-state level of government debt (see Chamley [1985]). Rather, steady-state debt is a function of the initial level of debt, b_0 , and the entire transition path of taxes and spending from $t = 0$ until the steady state is reached. As an alternative to performing this difficult computation, we follow the approach of Lucas (1990) and simply choose the level of steady-state debt to reflect a debt-to-GNP ratio consistent with the data. We assume that b_0 and the transition path are set such that the government budget constraint is satisfied in present-value terms. Data on U.S. federal debt held by the public are from *Federal Debt and Interest Costs*, Congressional Budget Office, 1993, table A-2.

resulting steady-state tax on capital is 0.31. This value of τ_k approximates the average effective corporate tax rate in the United States from 1947 to 1980, as estimated by Jorgenson and Sullivan (1981).¹⁰ The steady-state tax on labor (τ_h) turns out to be 0.25. This is close to the average marginal tax rate on labor income from 1947 to 1983, as estimated by Barro and Sahasakul (1986). The U.S. tax rate estimates can be viewed as summarizing the various elements of the tax code that impact the behavior of agents. These include not only the statutory rate, but also the many types of exemptions, deductions, credits, and allowances.

The monopoly power parameter χ is chosen such that the aggregate production technology demonstrates constant returns to scale. Given the values chosen for θ_1 and θ_2 , a value of $\chi = 0.91$ yields $y_t = \exp(z_t) k_t^{0.34} h_t^{0.66}$. The capital depreciation rate δ ($= 0.07$) is consistent with values commonly used in the RBC literature. Together with the values of β and θ_1 , this depreciation rate implies a steady-state ratio of capital to output ranging from 2.4 (under the distortionary tax structure) to 2.8 (under the lump-sum tax structure), and a ratio of investment to output ranging from 0.17 to 0.20. The corresponding average ratios for the U.S. economy from 1947 to 1992 are 2.58 and 0.21. The process governing technology shocks is estimated using annual data from 1947 to 1992. The series for z_t was constructed by computing the changes in output not accounted for by changes in the productive inputs.¹¹ The parameter estimates, $\rho_z = 0.85$ and $\sigma_\varepsilon = 0.015$, are close to those estimated by other studies using annual data, such as Benhabib and Jovanovic (1991).

IV. Simulation Results

In this section, we describe the model's predictions for the behavior of fiscal policy over the business cycle and provide a comparison with U.S. data. The simulation results are shown in

■ **10** Higher profit levels imply a higher steady-state tax on capital in our model. When profits are zero ($\theta_1 + \theta_2 = 1$), the optimal steady-state tax on capital is zero. If a separate profits tax were available, the government would choose to tax profits at 100 percent and other capital income at 0 percent in the steady state (see footnote 1).

■ **11** The production function residual was measured as $z_t = \ln GNP_t - 0.34 \ln k_t - 0.66 \ln h_t$. The private capital stock k_t is defined as fixed private capital + stock of consumer durables + residential capital from *Fixed Reproducible Tangible Wealth in the United States*. U.S. Department of Commerce, 1993. Real GNP and the labor input ($h_t = \text{LHOURS}$) are from Citibase.

tables 1–2 and figures 1–5. Note that the distortionary tax structure makes predictions for a larger set of variables than does the lump-sum tax structure. The additional variables are the stock of real government debt, b_t , and the average marginal tax rates on labor and capital income, τ_{hl} and τ_{kl} .¹²

In comparison to the full sample of U.S. data from 1947 to 1992, both tax structures underpredict the standard deviation of output (y_t), consumption (c_t), government expenditures (g_t), and hours (h_t), but overpredict the standard deviation of investment (x_t). Since we employ a general-equilibrium framework, the behavior of one variable cannot be viewed in isolation, because it is linked by the equilibrium conditions to the behavior of other variables in the model. For example, the low variability of output is linked to the low variability of hours, because the production technology is labor intensive.¹³ Likewise, the low variability of consumption is linked to the high variability of investment, because changes in household saving (which correspond to changes in investment) act as a buffer against earnings shocks, thereby allowing households to smooth their consumption over the business cycle. Consistent with standard RBC models (see Kydland and Prescott [1982]), both model versions capture the fact that output is more variable than consumption, but less variable than investment, over the U.S. business cycle.

In comparison to the lump-sum tax structure, the distortionary tax structure displays a lower standard deviation of output and hours and a higher variability of investment relative to output. This behavior can be traced to the movement of the optimal distortionary tax rates on labor and capital income. Specifically, the optimal labor tax is procyclical (positively correlated with output), while the optimal capital tax is countercyclical. The procyclical labor tax operates to smooth households' after-tax income from labor. For example, a positive technology shock (which shifts the production frontier outward and raises the real wage) is accompanied by an increase in τ_{hl} . The higher tax rate tends to offset the higher real wage and thus provides households with an implicit insurance mechanism against

■ **12** In all figures, model variables are the realizations from a single simulation (based on randomly drawn shocks) to compare volatility and persistence properties with the corresponding U.S. variables. There is no intention to predict the actual time path of U.S. variables.

■ **13** This can be readily observed from the aggregate production function (equation [8a]), where the coefficient on the labor input, $\alpha_2 = 0.66$, is nearly double the coefficient on the capital input, $\alpha_1 = 0.34$.

TABLE 1

Business Cycle Statistics for Models and the U.S. Economy

Variables	Standard Deviation (percent)			
	Distortionary Tax Structure ^a	Lump-Sum Tax Structure ^a	U.S. Economy ^b	
			1947-92	1954-92
y_t	2.11	2.39	2.46	2.16
c_t	0.69	0.96	1.14	1.19
g_t	1.66	1.00	6.45	3.04
k_t	1.05	1.03	0.75	0.74
x_t	9.10	9.00	5.96	6.14
b_t	1.19	1.60	1.76	1.86
y_t/b_t	1.09	0.96	1.45	0.90
b_t	2.72	—	4.54	4.80
Tax rates				
τ_{bt}	1.71	—	6.42	4.08
τ_{kt}	13.91	—	17.76	19.28
Rev/y_t	3.61	1.62	2.65	1.88
Contemporaneous Correlation with Output				
Variables	Distortionary Tax Structure ^a	Lump-Sum Tax Structure ^a	U.S. Economy ^b	
			1947-92	1954-92
c_t	0.67	0.89	0.71	0.87
g_t	0.98	0.85	0.62	0.40
k_t	0.03	-0.01	0.62	0.63
x_t	0.97	0.97	0.69	0.89
b_t	0.93	0.96	0.81	0.91
y_t/b_t	0.92	0.89	0.71	0.52
b_t	0.03	—	0.23	0.36
Tax rates				
τ_{bt}	0.97	—	0.39	-0.10
τ_{kt}	-0.93	—	0.08	-0.15
Rev/y_t	-0.91	-0.95	0.39	0.11

a. Model statistics are means over 100 simulations, each 46 periods long, after dropping the first 50 periods. The symbol Rev/y_t is the economywide average tax rate, defined as total tax revenue as a fraction of output.

b. The following quarterly series from Citibase were annualized before computing the statistics: y_t = GNPQ, c_t = GCNQ + GCSQ (nondurables + services), g_t = GGEGQ, b_t = LHOURLS (household survey), and y_t/b_t = GNPQ/LHOURLS. The series for x_t is business fixed investment + consumer durable expenditures + residential investment. The series for k_t is fixed private capital + stock of consumer durables + residential capital. Both x_t and k_t are annual series from *Fixed Reproducible Tangible Wealth in the United States*, U.S. Department of Commerce, 1993. The series for b_t is federal debt held by the public from *Federal Debt and Interest Costs*, Congressional Budget Office, 1993, table A-2, where nominal debt has been converted into real debt by dividing by the GNP deflator for each year. Rev/y_t is total government receipts (federal, state, and local) as a fraction of GNP from *Economic Report of the President*, 1991, 1994, table B-80. Data on average marginal tax rates do not extend over the full sample: τ_{bt} is from Barro and Sahasakul (1986) for 1947-83, and τ_{kt} is from Jorgenson and Sullivan (1981), table 11, for 1947-80. NOTE: Before computing the statistics, all series were logged and detrended using the Hodrick-Prescott filter (see Prescott [1986]). The smoothing parameter for the filter was set at 100, since all data are at annual frequency.

SOURCE: Authors' calculations.

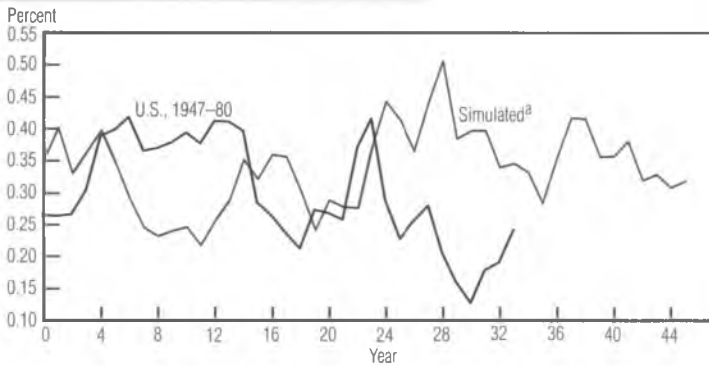
variability in the *after-tax* wage $(1 - \tau_{bt}) w_t$. Since labor supply decisions depend on the quantity $(1 - \tau_{bt}) w_t$ (see equation [4b]), a lower variability in the after-tax wage leads to a lower variability in hours worked. With a labor-intensive production technology, this also leads to lower variability in output.

Zhu (1992) shows theoretically that the cyclical behavior of the optimal labor tax depends on the degree of risk aversion (or curvature) exhibited by the household utility function. The optimal labor tax is procyclical for low-risk-aversion (less curved) utility functions, such as the logarithmic case used here, but countercyclical for high-risk-aversion (more curved) functions. Chari, Christiano, and Kehoe (1994) provide a quantitative demonstration of this result. In general, the level of risk aversion determines the amount by which households are willing to adjust their labor supply in response to a change in the real wage. With low risk aversion, the substitution effect of an increase in w_t (caused by a positive technology shock) results in a relatively large increase in labor hours. The government takes advantage of this greater willingness to work by *raising* the tax on labor, thereby collecting additional revenue, but still allowing an increase in labor to spur output during this period of high labor productivity. With high risk aversion, however, the substitution effect is much smaller; that is, households are less willing to increase their labor supply in response to the higher real wage. The government's optimal response now is to *lower* the tax rate on labor. This stimulates labor supply in order to boost output while labor productivity is high. Our results are consistent with the findings of these researchers.

The capital tax in the model moves countercyclically and displays high variability relative to the labor tax (see table 1 and figures 1-2). This serves to increase the variability of household investment relative to output under the distortionary tax structure. Ordinarily, a positive technology shock raises the real rate of interest and motivates an increase in investment because the rate of return becomes more attractive. However, when a positive technology shock is accompanied by a *decrease* in τ_{kt} , the *after-tax* return on investment becomes even more appealing, leading to a larger rise in investment. From the government's perspective, a countercyclical capital tax is optimal because it serves as an efficient means of absorbing shocks to the government's budget constraint. These shocks are caused by changes in the size of the tax base over the the business cycle. For example, a positive technology shock generates more tax

FIGURE 1

Capital Tax Rates

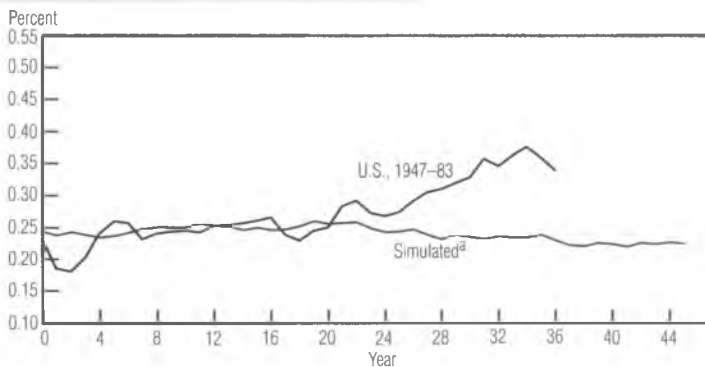


a. Distortionary tax structure.

SOURCES: Jorgenson and Sullivan (1981); and authors' calculations.

FIGURE 2

Labor Tax Rates

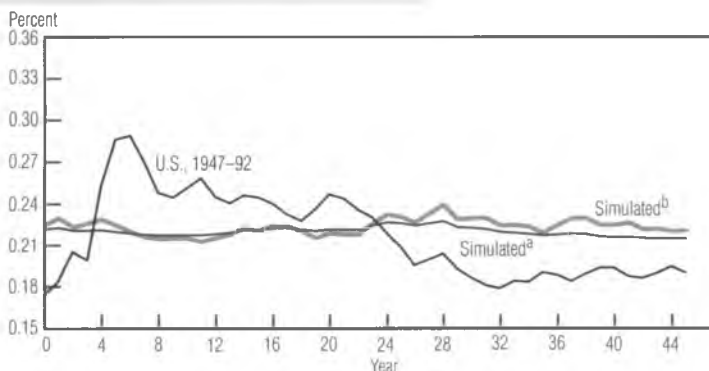


a. Distortionary tax structure.

SOURCES: Barro and Sahasakul (1986); and authors' calculations.

FIGURE 3

Government Expenditures as a Fraction of GNP



a. Distortionary tax structure.

b. Lump-sum tax structure.

SOURCES: Citibase; and authors' calculations.

revenue because GNP and household incomes (the tax base) increase. This motivates a reduction in τ_{kl} because government spending requirements can be met using a lower tax rate. A similar argument holds in reverse for the case of a negative technology shock. Absorbing shocks mainly by changes in τ_{kl} , as opposed to changes in τ_{bl} , is efficient because the capital stock cannot be quickly adjusted in response to a change in the capital tax. In contrast, the household can instantaneously adjust labor supply in response to a change in the labor tax. The shock-absorbing feature of τ_{kl} allows the government to maintain a very smooth time series for g_t/y_t , as compared to the lump-sum tax structure (see figure 3).¹⁴

We experimented with varying the level of monopoly profits by adjusting the values of the parameters θ_1 , θ_2 , and χ . In general, we found that as profits declined, the standard deviation of τ_{kl} increased. The intuition for this result is straightforward. Recall that dividends (equal to after-tax profits) do not distort household decisions because profits are determined outside households' control. A lower level of profits implies a smaller and more elastic tax base for the capital tax. Consequently, larger changes in the tax rate are needed to produce the same revenue effect when responding to technology shocks.

The model's prediction that the capital tax should display more variability than the labor tax is consistent with the U.S. tax-rate estimates we have chosen for comparison.¹⁵ Note, however, that the correlation coefficients between U.S. tax rates and real GNP display a change in sign, depending on the sample period. The labor tax and the capital tax are weakly *procyclical* using data on average marginal tax rates that begin in 1947, but weakly *countercyclical* for data that begin in 1954. The model, on the other hand, predicts a strongly procyclical labor tax and a strongly countercyclical capital tax. Thus, there is a sharp negative correlation

■ 14 The optimality of using a state-contingent capital tax to absorb budget shocks has been shown previously by Judd (1989) and Chari, Christiano, and Kehoe (1994). Our quantitative results are not directly comparable because Judd does not explicitly model household behavior, and Chari, Christiano, and Kehoe employ a different decentralization scheme for τ_{kl} and r_{bt} .

■ 15 The figures display the tax-rate series *before* detrending. For quantitative comparisons (table 1), detrending is necessary because the U.S. labor tax displays a distinct upward trend, while the U.S. capital tax displays a downward trend. These trends have no counterpart in the model. The trend in τ_{bl} is possibly linked to the phenomenon of "bracket creep," which existed before tax schedules were indexed for inflation in 1985. Auerbach and Poterba (1988) argue that the downward trend in τ_{kl} is due to increasingly generous investment tax credits and accelerated depreciation schedules.

FIGURE 4

Total Tax Revenue as a Fraction of GNP



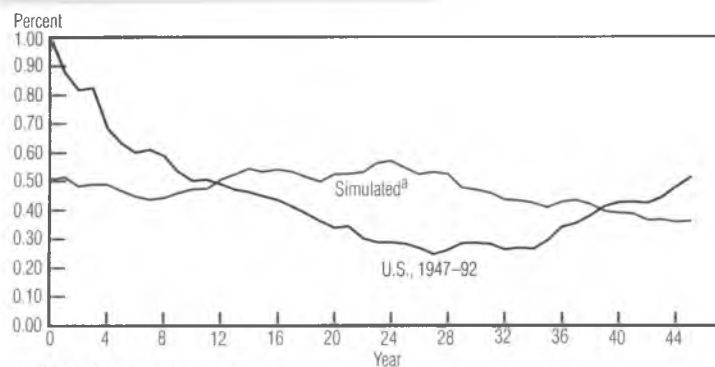
a. Distortionary tax structure.

b. Lump-sum tax structure.

SOURCES: *Economic Report of the President*, 1991, 1994; and authors' calculations.

FIGURE 5

Government Debt as a Fraction of GNP



a. Distortionary tax structure.

SOURCES: Congressional Budget Office, *Federal Debt and Interest Costs*, 1993; and authors' calculations.

between τ_{kt} and τ_{bt} in the model, while the corresponding correlation in the data is weakly positive.¹⁶ Figure 4 displays the economywide average tax rate, defined as total tax revenue as a fraction of output. This rate is countercyclical in both versions of the model, but weakly procyclical in the U.S. data. Thus, the model does not capture some important features of U.S. tax-rate movements. This highlights the difficulty of summarizing the entire U.S. tax code using only one or two broadly defined rates.

The predicted standard deviations for g_t are substantially lower than the U.S. value of 6.45 percent computed using the full sample. Starting the sample in 1954 to avoid the influence of the Korean War reduces the standard deviation

cent, which is much closer to model predictions. Although we have no theoretical justification for excluding the Korean War years (since we include the Vietnam War), the fact that our model contains only one type of shock limits its ability to explain large movements associated with a war. Incorporating an additional shock to households' preference for public goods to simulate high demand during wars would increase the variability of g_t in the model (see Lansing [1994]). Both model versions capture the procyclical behavior of government spending in annual U.S. data, but the correlations from the model are much stronger than observed in the data. When the years prior to 1954 are excluded, the correlation between government spending and real output in the U.S. data drops from 0.62 to 0.40, worsening the comparison with the model.

In the distortionary tax structure, government debt is essentially acyclical (it behaves similarly to capital in this respect), but is less variable than U.S. government debt (see figure 5). We experimented with an alternate version of this model in which the government is required to balance its budget each period. For this experiment, we retained the decentralization scheme described in section II, whereby the government is required to satisfy the certainty version of equation (4c). Qualitatively, the results are similar to those reported in table 1. However, the variability of the labor tax goes up, since government debt is no longer available to help cushion budget shocks. The insurance effect of the procyclical labor tax thus becomes more significant, leading to lower variability in hours and output. Interestingly, our model implies that a balanced-budget amendment can help smooth business cycle fluctuations, provided the government sets tax rates in the manner we have described.

The lump-sum tax structure does a reasonably good job of matching the relative standard deviations of hours and the real wage, where the real wage is measured by average labor productivity y_t/h_t (see table 2). This behavior is typical of standard RBC models with indivisible labor (see Hansen [1985]). In the distortionary tax structure, however, the standard deviation of hours relative to the real wage is too low, despite the specification of indivisible labor. The insurance effect of the procyclical labor tax

■ 16 In the U.S. data, the correlation coefficient between (logged and detrended) τ_{bt} and τ_{kt} equals 0.36 from 1947 to 1980, the period for which estimates of both tax rates are available. For the years 1954 to 1980, the correlation coefficient is 0.34. In the model, the correlation coefficient is -0.97 .

TABLE 2

Comparison of Labor Market Statistics

Variables	Distortionary Tax Structure ^a	Lump-Sum Tax Structure ^a	U.S. Economy ^b	
			1947-92	1954-92
$\sigma_b / \sigma_{y/b}$	1.09	1.67	1.22 2.10	2.07 2.43
corr ($b, y/b$)	0.72	0.72	0.16 -0.02	0.13 -0.21

a. Model statistics are means over 100 simulations, each 46 periods long, after dropping the first 50 periods.

b. The first number denotes hours worked from the household survey in Citibase (LHOURS), and the second denotes hours worked from the establishment survey (LPMHU).

NOTE: Before computing the statistics, all series were logged and detrended using the Hodrick-Prescott filter (see Prescott [1986]). The smoothing parameter for the filter was set at 100, since all data are at annual frequency.

SOURCE: Authors' calculations.

is responsible for the reduced variability of hours. Again, we note that the model includes only one type of shock. Aiyagari (1994) uses a variance decomposition analysis to argue that the behavior of hours in U.S. data is driven by multiple shocks.

Finally, the predicted correlations between hours and productivity in table 2 are much higher than the corresponding U.S. values. Braun (1994) shows that an RBC model with *exogenous* stochastic tax rates is capable of matching both of the U.S. labor market statistics in the table. Our simulations show that a model with endogenous tax rates can produce very different results.¹⁷

V. Conclusion

We have constructed a model that combines elements from the theory of optimal public finance with an RBC view of aggregate fluctuations. Our aim is to develop a framework that is useful for carrying out realistic policy experiments with regard to both the structure of the U.S. tax system and the composition and financing of government expenditures. In two related papers (see Guo and Lansing [1994a, 1994b]), we employ models similar to

this one to explore the welfare effects of various tax structures and find that these effects can be quite dramatic. In this paper, our model meets with varying degrees of success in matching the observed behavior of tax rates, government spending, and aggregate economic variables in the U.S. economy. Nonetheless, this exercise is useful in that it provides information on how models of government fiscal policy might be improved.

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■ 17 See Chari, Christiano, and Kehoe (1994) for a more extensive comparison between an RBC model with exogenous stochastic tax rates and one with optimal Ramsey tax rates.

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