Real-Time Estimation of Trend Output and the Illusion of Interest Rate Smoothing*

Kevin J. Lansing

Senior Economist
Federal Reserve Bank of San Francisco

Empirical estimates of the Federal Reserve's policy rule typically find that the regression coefficient on the lagged federal funds rate is around 0.8 and strongly significant. One economic interpretation of this result is that the Fed intentionally "smoothes" interest rates, i.e., policymakers move gradually over time to bring the current level of the funds rate in line with a desired level that is determined by consideration of recent economic data. This paper develops a small forward-looking macroeconomic model where in each period, the Federal Reserve constructs a current, or "real-time," estimate of trend output by running a regression on past output data. Using the model as a data-generating mechanism, I show that efforts to identify the Fed's policy rule using final data (as opposed to real-time data) can create the illusion of interest rate smoothing behavior when, in fact, none exists. In particular, I show that the lagged federal funds rate can enter spuriously in final-data policy rule regressions because it helps pick up the Fed's serially correlated real-time measurement errors which are not taken into account by the standard estimation procedure. In model simulations, I find that this misspecification problem can explain as much as one-half of the apparent degree of "inertia" or "partial adjustment" in the U.S. federal funds rate.

1. Introduction

The Federal Reserve conducts monetary policy primarily through open market operations that influence the overnight interest rate on borrowed reserves among U.S. banks. The overnight interest rate is known as the federal funds rate. The target level for the federal funds rate is set by the Federal Open Market Committee (FOMC), which meets eight times per year. In deciding the appropriate level of the funds rate, members of the FOMC carefully consider the most recent economic data and the implications for the economy going forward.

Given the way in which monetary policy is actually conducted, it is often useful to think about Federal Reserve behavior in terms of a "reaction function" or a "policy rule" that describes how the federal funds rate responds to key macroeconomic variables. An example of such a rule is the one suggested by Taylor (1993). According to the Taylor rule, the appropriate level of the funds rate is determined by a particular weighted combination of the deviation of inflation from a long-run target inflation rate and the "output gap," i.e., the difference between real output and a measure of trend (or potential) output. Interestingly, the path of the U.S. federal funds rate largely appears to conform to the recommendations of the Taylor rule starting in the mid- to late 1980s and extending into the 1990s. This observation has led to a large number of empirical studies that attempt to estimate the Fed's policy rule directly from U.S. data.

Motivated by the form of the Taylor rule, empirical studies of the Fed's policy rule typically regress the federal funds rate on a set of explanatory variables that includes the inflation rate (or a forecast of future inflation) and a measure of real economic activity such as the output gap. Many of these studies also include the lagged value of the federal funds rate as an additional explanatory variable. This feature turns out to greatly improve the empirical fit of the estimated rule. Using quarterly U.S. data, the regression coefficient on the lagged federal funds rate is generally found to be around 0.8 and strongly significant.[1] One economic interpretation of this result is that the Fed intentionally "smoothes" interest rates, i.e., policymakers move gradually over several quarters to bring the current level of the funds rate in line with a desired level that is determined by consideration of recent economic data. Under this view, the magnitude of the regression coefficient on the lagged

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1. See, for example, Amato and Laubach (1999), Clarida, et al. (2000), and Rudebusch (2002).

funds rate governs the degree of "inertia" or "partial adjustment" in Fed policy decisions.[2]

Given the apparent degree of interest rate smoothing in quarterly U.S. data, a large amount of research has been devoted to understanding why the Federal Reserve might wish to engage in such behavior.[3] Sack and Weiland (2000) review this research and identify three main arguments that could help explain the apparent gradual response of Fed policymakers to quarterly changes in inflation and the output gap. These are (1) forward-looking expectations, (2) uncertainty about economic data that are subject to revision, and (3) uncertainty about the structure of the economy and the transmission mechanism for monetary policy.

In an economy with forward-looking agents, policymakers can influence current economic activity by affecting agents' expectations about *future* policy actions. If agents are convinced that an initial change in the federal funds rate will be followed by additional changes in the same direction (as policymakers gradually adjust the funds rate toward the desired level), then the initial policy move will have a larger impact on agents' decisions. This feature of the economy allows policymakers to achieve their stabilization objectives without having to resort to large, abrupt policy moves, which may be viewed as undesirable because they increase interest rate volatility.[4] Consideration of uncertainty also favors gradual adjustment because policymakers tend to be cautious. Aggressive policy actions are generally resisted because they can lead to severe unintended consequences if the beliefs that motivated such actions later prove to be unfounded.

Without disputing the potential benefits of interest rate smoothing laid out in the above arguments, this paper shows that efforts to identify the Fed's policy rule using regressions based on final (or ex post revised) data can create the illusion of interest rate smoothing behavior when, in fact, none exists. In particular, I show that the lagged federal funds rate can enter spuriously in final-data policy rule regressions because it helps pick up the Fed's serially correlated real-time measurement errors which are not taken into account by the standard estimation procedure.

The framework for my analysis is a small forward-looking macroeconomic model where in each period the Federal Reserve constructs a current, or "real-time," estimate of the level of potential output by running a regression on past output data. The Fed's perceived output gap (the difference between actual output and the Fed's real-time estimate of potential output) is used as an input to the monetary policy rule, while the true output gap influences aggregate demand and inflation.

As in Lansing (2000), I allow for the possibility that true potential output may undergo abrupt shifts in level and/or slope which are unknown to Fed policymakers until some years later. In the model, true potential output is calibrated to match a segmented linear trend fit to U.S. data on real GDP. I allow for two abrupt trend shifts: the first captures the well-documented productivity slowdown of the early 1970s while the second captures the postulated arrival of the so-called "new economy" in the mid-1990s, which is thought by some to be characterized by faster trend productivity growth.[5] Initially, Fed policymakers interpret these trend shifts to be cyclical shocks but their regression algorithm allows them to discover the truth gradually as the economy evolves by assigning more weight to the recent data.

Using the model as a data-generating mechanism, I produce artificial data on interest rates, inflation, and real output for the case where Fed policymakers employ a Taylor-type rule with no interest rate smoothing whatsoever. I then take the perspective of an econometrician who uses these data to estimate the Fed's policy rule. I consider two possible misspecifications of the econometrician's regression equation. First, the econometrician uses a final-data potential output series instead of the Fed's real-time potential output estimates. To keep things simple, I endow the econometrician with full knowledge of the true potential output series defined by the segmented linear trend. Hence, the econometrician's final-data potential output series coincides exactly with the true series (but differs from the Fed's real-time estimates). Second, the econometrician may adopt the wrong functional form for the policy rule, i.e., one that differs from the Taylor-type rule that Fed policymakers are actually using in the model. Specifically, I consider the case where the econometrician includes an additional lag of the output gap in the regression equation. The additional lag would be appropriate if the econometri-

2. The concept of interest rate smoothing is often linked to the idea that Fed policymakers adjust the funds rate in a series of small steps and reverse course only at infrequent intervals. Rudebusch (2002) notes that while this concept of interest rate smoothing applies to federal funds rate movements over the course of several weeks or months, it does not necessarily imply a large regression coefficient on the lagged funds rate at quarterly frequency.

3. The central banks of other countries also appear to exhibit interest rate smoothing behavior. For some details, see Lowe and Ellis (1997) and Srour (2001).

4. For a formal theoretical argument along these lines, see Woodford (1999).

5. Oliner and Sichel (2000) present evidence of a pickup in measured U.S. productivity growth after 1995 that appears to be linked to spending on information technology. Gordon (2000) argues that a proper analysis of the productivity data does not support the views of the new economy enthusiasts.

cian believed that Fed policymakers were responding to the deviation of nominal income growth from a long-run target growth rate.

Over the course of 1,000 model simulations, I find that the econometrician almost always obtains a positive and strongly significant regression coefficient on the lagged federal funds rate, even though the Fed in the model is not engaging in any interest rate smoothing. The average point estimate of the spurious regression coefficient is around 0.3 or 0.4, depending on the econometrician's sample period and rule specification. The intuition for this result is straightforward. Since the Fed's algorithm for estimating potential output assigns more weight to recent data, the end-of-sample estimate can undergo substantial changes as new observations arrive—even without a trend shift in the underlying economy. The algorithm gives rise to serially correlated real-time measurement errors that influence the period-by-period setting of the federal funds rate. By ignoring these errors, the econometrician's final-data regression equation is subject to a missing variable problem. The inclusion of the lagged funds rate helps compensate for the problem by acting as a proxy for the missing error terms.

The simulations show that failure to account properly for the Fed's real-time perceptions about potential output can explain as much as one-half of the apparent degree of inertia in the U.S. federal funds rate. This finding complements recent work by Rudebusch (2002), who uses evidence from the term structure of U.S. interest rates to reject the hypothesis of a large degree of monetary policy inertia. Under the assumption that longer-term interest rates are governed by agents' rational expectations of future short-term rates, Rudebusch shows that a coefficient of 0.8 on the lagged federal funds rate is not consistent with U.S. term structure data. A smaller coefficient on the lagged funds rate of, say, 0.4 cannot be rejected, however. Rudebusch draws on a variety of qualitative evidence from historical episodes to argue that "quarterly interest rate smoothing is a very modest phenomenon in practice."

Finally it should be noted that some recent empirical studies have made serious efforts to take into account the Fed's real-time information set when estimating policy rules directly from U.S. data. Examples include the studies by Orphanides (2001), Perez (2001), and Mehra (2001) who employ reconstructed historical data that is intended to capture the information available to Fed policymakers at the time policy decisions actually were made. Orphanides (2001) and Perez (2001) continue to find a large and statistically significant coefficient on the lagged federal funds rate even when policy rules are regressed on the reconstructed real-time data, while Mehra (2001) does not. In particular, Mehra (2001) shows that the lagged funds rate actually may be picking up the Fed's real-time response to a "smoothed" inflation rate which is defined by a four-quarter moving average of the quarterly inflation rate.

A drawback of the reconstruction approach is that we cannot know for sure what method was being used by Fed policymakers to estimate potential output in real time. Indeed, each of the three studies mentioned above adopts a different method for defining the Fed's real-time estimate of potential output.[6] Another drawback of the reconstruction approach is that we cannot know the exact form of the policy rule that was being used by Fed policymakers during a given period of history. The simulation-based approach adopted here avoids these drawbacks by conducting a controlled scientific experiment where we have full knowledge of all factors that govern the real-time decisions of Fed policymakers.

The remainder of the paper is organized as follows. Section 2 describes the model that is used to generate the artificial data. Section 3 describes the simulation procedure. Section 4 presents the results of the simulations. Section 5 concludes.

2. The Model

In this paper, the economic model serves as a data-generating mechanism for the policy rule regressions that are the main focus of the analysis. I use a small forward-looking macroeconomic model adapted from Lansing (2000). The details are contained in Box 1. The model consists of: (1) an aggregate demand equation that links real economic activity to the level of the real interest rate, (2) an equation that describes how true potential output evolves over time, (3) a short-run Phillips curve that links inflation to the level of real economic activity, (4) a term structure equation that links the behavior of short- and long-term interest rates, and (5) a monetary policy rule that describes how the federal funds rate responds to inflation and real economic activity. The model is quite tractable and has the advantage of being able to reproduce the dynamic correlations among U.S. inflation, short-term nominal interest rates, and deviations of real GDP from trend. Lansing (2000) shows that the model also can replicate some of the key low-frequency movements in U.S. inflation over the past several decades.

6. Orphanides (2001) assumes that real-time potential output is defined by the Federal Reserve staff's Q^* series which is constructed as a segmented linear trend linked to Okun's law. Perez (2001) assumes that real-time potential output is defined by the Hodrick-Prescott (1997) filter. Mehra (2001) assumes that real-time potential output is defined by a log-linear trend fitted to observations of past output.

Box 1
Details of the Model

The equations that describe the model are as follows:

Aggregate Demand Equation

$$(1)\qquad y_t - \overline{y}_t = a_1\left(y_{t-1} - \overline{y}_{t-1}\right) + a_2\left(y_{t-2} - \overline{y}_{t-2}\right) + a_r\left(r_{t-1} - \overline{r}\right) + v_t,\ v_t \sim N\left(0, \sigma_v^2\right),$$

where y_t is the logarithm of real output (GDP), $\overline{y}_t$ is the logarithm of true potential output, r_{t-1} is the lagged value of the ex ante long-term real interest rate, and v_t is a shock to aggregate demand that may arise, for example, due to changes in government purchases. The true output gap is given by $y_t - \overline{y}_t$. In steady state, the output gap is 0 which implies that $\overline{r}$ is the steady-state real interest rate.

True Potential Output

$$(2)\qquad \overline{y}_t = \begin{cases} c_0 + \mu_0 \cdot t & \text{for } t_0 \le t \le t_1, \\ c_1 + \mu_1 \cdot t & \text{for } t_1 < t \le t_2, \\ c_2 + \mu_2 \cdot t & \text{for } t > t_2, \end{cases}$$

where c_i and μ_i for $i = 0, 1, 2$ represent the intercept and slope terms for a segmented linear trend with breakpoints at t_1 and t_2.

Short-run Phillips Curve

$$(3)\qquad \pi_t = \tfrac{1}{2}(\pi_{t-1} + E_{t-1}\pi_t) + \gamma\left(y_{t-1} - \overline{y}_{t-1}\right) + z_t,\ z_t \sim N\left(0, \sigma_z^2\right),$$

where π_t is the fully observable inflation rate defined as the log difference of the price level (the GDP price index), E_{t-1} is the expectation operator conditional on information available at time $t-1$, and z_t is a cost-push shock. The steady-state version of equation (3) implies that there is no steady-state trade-off between inflation and real output.

Term Structure Equation

$$(4)\qquad r_{t-1} = \tfrac{1}{2}E_{t-1}\sum_{i=0}^{1}\left(i_{t-1+i} - \pi_{t+i}\right) = \tfrac{1}{2}\left(i_{t-1} - E_{t-1}\pi_t + E_{t-1}i_t - E_{t-1}\pi_{t+1}\right),$$

where i_t is the one-period nominal interest rate (the federal funds rate). Equation (4) summarizes the expectations theory of the term structure for an economy where the "long-term" interest rate corresponds to a two-period Treasury security (the six-month T-Bill). Private-sector agents use their knowledge of the Fed's policy rule to compute the expectation $E_{t-1}i_t$. In steady state, equation (4) implies the Fisher relationship: $\overline{i} = \overline{r} + \overline{\pi}$.

Federal Reserve Policy Rule

$$(5)\qquad i_t^* = \overline{r} + \overline{\pi} + g_\pi\left(\pi_{t-1} - \overline{\pi}\right) + g_y\left[y_{t-1} - \overline{x}_{t-1} - \phi\left(y_{t-2} - \overline{x}_{t-2}\right)\right],$$

$$(6)\qquad i_t = \rho i_{t-1} + (1-\rho)\, i_t^*,$$

where $\overline{\pi}$ is the Fed's long-run inflation target which determines the steady-state inflation rate. The symbol $\overline{x}_t$ represents the Fed's real-time estimate of $\overline{y}_t$. This estimate is constructed by applying a regression algorithm to the historical sequence of real output data $\{y_s\}_{s=t_0}^{s=t}$ which is fully observable. The symbol i_t^* represents the desired (or target) level of the federal funds rate. The parameter $0 \le \rho \le 1$ governs the degree of inertia (or partial adjustment) in the funds rate.

Equations (5) and (6) capture most of the rule specifications that have been studied in the recent monetary policy literature. A simple version of the original Taylor (1993) rule can be represented by $\rho = 0$, $g_\pi = 1.5$, $g_y = 0.5$, and $\phi = 0$.[a] Taylor (1999) considers a modified version of this rule which is characterized by a stronger response to the output gap, i.e., $g_y = 1.0$ rather than $g_y = 0.5$.[b] In the appendix, I show that a nominal income growth rule can be obtained by setting $g_\pi = g_y$ with $\phi = 1$.

When $g_\pi > 1$, the desired funds rate i_t^* moves more than one-for-one with inflation. This feature is generally viewed as desirable because it tends to stabilize inflation; any increase in the inflation rate brings about a larger increase in the desired nominal funds rate which will eventually lead to a higher *real* interest rate. A higher real rate restrains aggregate demand and thereby helps to push inflation back down.[c]

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[a] The original Taylor (1993) rule assumes that the funds rate responds to the average inflation rate over the past four quarters, whereas equations (5) and (6) imply that the funds rate responds to the inflation rate in the most recent quarter only.

[b] Lansing and Trehan (2001) consider the issue of whether either version of the Taylor rule can be reconciled with optimal discretionary monetary policy.

[c] For additional details, see Taylor (1999) and Clarida, et al. (2000).

Private-sector agents in the model are completely informed at all times regarding the level of true potential output. This can be justified in one of two ways: (1) the private sector consists of a large number of identical firms, each of which knows its own productive capacity, or (2) the process of aggregating over the distribution of firms yields an economy-wide description that is observationally equivalent to (1). Private-sector agents have rational expectations; they know the form of the monetary policy rule and the Fed's estimate of potential output which is used as an input to the rule.

True potential output in the model is trend stationary but subject to infrequent shifts in level and/or slope. Perron (1989) shows that standard statistical tests cannot reject the hypothesis of a unit root in U.S. real output data when the true data-generating mechanism is one of stationary fluctuations around a deterministic trend with infrequent shifts. More recently, Dolmas, et al. (1999) argue that U.S. labor productivity is more accurately modeled as a deterministic trend with a sudden change in level and slope around 1973, rather than as a unit root process. For simplicity, the model abstracts from any direct theoretical link between the slope of true potential output (which measures the economy's trend growth rate) and the value of the steady-state real interest rate. Even without assumption, however, a sudden unanticipated change in the *level* of potential output would have no theoretical implications for the value of the steady-state real rate.

Following the framework of Clarida, et al. (2000), I use the symbol i_t^* to represent the desired (or target) level of the federal funds rate that is determined by policymakers' consideration of economic fundamentals. The relevant fundamentals include: (1) the level of the steady-state real interest rate, (2) the deviation of recent inflation from the Fed's long-run target rate, and (3) the gap between recent output and the Fed's real-time estimate of potential output. The model's policy rule specification allows for the possibility that the Fed does not immediately adjust the funds rate to the desired rate but instead engages in "interest rate smoothing" whereby the current federal funds rate i_t is moved in the direction of the desired rate i_t^* over time. The parameter ρ is used here to represent the degree of inertia (or partial adjustment) in the funds rate. Each period the Fed moves the actual funds rate by an amount equal to the fraction $(1-\rho)$ of the distance between the desired rate and the actual rate.[7] When $\rho = 0,$ the adjustment process is immediate; the Fed sets the actual rate equal to the desired rate each period.

Fed policymakers in the model cannot directly observe true potential output or the shocks hitting the economy. The hidden nature of the shocks is crucial because it prevents policymakers from using any knowledge they may have about the structure of the economy to back-solve for true potential output. The assumption of asymmetric information between the private sector and the Fed is consistent with some recent papers that investigate the performance of alternative policy rules in environments where the output gap that appears in the rule is subject to exogenous stochastic shocks. These shocks are interpreted as "noise" or "measurement error."[8] Unlike these exercises, however, the measurement error in this model is wholly endogenous—it depends on the structure of the economy, the form of the policy rule, and the regression algorithm used by the Fed to construct the real-time potential output series.

The policy rule specification implies that the Fed reacts only to lagged variables and not to contemporaneous variables. This feature addresses the point made by McCallum (1999) that policy rules should be "operational," i.e., rules should reflect the fact that policy decisions often must be made before economic data for the current quarter become available. Finally, as in most quantitative studies of monetary policy rules, the model abstracts from the zero lower bound on nominal interest rates.

2.1. Real-Time Estimate of Potential Output

Fed policymakers in the model construct a current, or "real-time," estimate of potential output by running a regression on the historical sequence of real output data. The regression algorithm can be viewed as part of the Fed's policy rule. In choosing an algorithm, I assume that policymakers wish to guard against the possibility that potential output may undergo trend shifts.[9] This feature is achieved through the use of an algorithm that assigns more weight to recent data in constructing the end-of-sample estimate. The result is a flexible trend that can adapt to shifts in true potential output. The Fed's real-time potential output series is updated each period so that policymakers continually revise their view of the past as new observations arrive.

The particular regression algorithm used here is known as the Hodrick-Prescott (HP) filter.[10] The HP filter minimizes the sum of squared differences between trend and

7. This can be seen by subtracting i_{t-1} from both sides of equation (6) to yield $i_t - i_{t-1} = (1-\rho)\left(i_t^* - i_{t-1}\right).$

8. See, for example, Orphanides, et al. (2000).

9. See Parry (2000) for some evidence that real-world policymakers guard against the possibility of trend shifts.

10. For details, see Hodrick and Prescott (1997).

the actual series, subject to a penalty term that constrains the size of the second differences.[11] Use of this algorithm introduces an additional parameter into the model, namely, the weight λ assigned to the penalty term. The value of λ controls the smoothness of the resulting trend. When $\lambda = 0$, the HP filter returns the original series with no smoothing whatsoever. As $\lambda \rightarrow \infty$, the HP filter returns an ordinary least squares (OLS) trend for interior points of a finite sample, but there can be significant distortions from OLS near the sample endpoints. When λ = 1,600, the HP filter applied to a quarterly series approximates a band-pass filter that extracts components of the data that are typically associated with business cycles or high-frequency noise, i.e., components with fluctuations between 2 and 32 quarters. Again, however, there may be significant distortions from the ideal band-pass filter near the sample endpoints.[12]

St-Amant and van Norden (1997) show that when λ = 1,600, the HP filter assigns a weight of 20 percent to observations at the end of the sample, whereas observations at the center of the sample receive no more than a 6 percent weight. Real-time estimates of potential output constructed using the HP filter may therefore undergo substantial changes as new observations arrive—even without a trend shift in the underlying economy or revisions to published data.[13] Orphanides and van Norden (1999) show that this problem arises with other real-time methods of trend estimation as well, but with varying degrees of severity. Unfortunately, the problem cannot be avoided because the future trajectory of the economy (which cannot be known in advance) turns out to provide valuable information about the current level of potential output.

In describing the HP filter, Kydland and Prescott (1990, p. 9) claim that the "implied trend path for the logarithm of real GNP is close to one that students of business cycles and growth would draw through a time plot of this series." One might argue that Fed policymakers could obtain a more accurate estimate of potential output by taking into account observations of other variables, such as inflation, or by solving an optimal signal extraction problem. I choose not to pursue these options here because their application hinges on the strong assumption that Fed policymakers possess detailed knowledge about key structural features of the economy such as the slope of the short-run Phillips curve or the distributions governing unobservable shocks. Given that simple univariate algorithms such as the HP filter are commonly used to define potential output in monetary policy research (see, for example, Taylor 1999), the idea that Fed policymakers would adopt similar techniques does not seem unreasonable.

2.2. Policy Rule Misspecification

An econometrician who uses final data to estimate the Fed's policy rule is implicitly assuming that the final-data version of potential output is equal to the Fed's real-time version of potential output. In the model, this assumption is false. The Fed's regression algorithm gives rise to real-time measurement errors that influence the period-by-period setting of the federal funds rate. By ignoring these errors, the econometrician's final-data estimation procedure is subject to a missing variable problem (for details, see Box 2). The Fed's real-time measurement errors turn out to be highly serially correlated in the quantitative simulations. In such an environment, the econometrician's estimate of the inertia parameter ρ will be biased upward relative to the true value because the lagged funds rate serves as a proxy for the missing error terms. This is an example of a well-known econometric problem originally analyzed by Griliches (1961, 1967). In particular, Griliches shows that the OLS estimate of a partial adjustment coefficient (such as ρ) will be biased upward relative to its true value if the econometrician ignores the presence of positive serial correlation in the error term.[14] Exploiting this idea, Rudebusch (2002) demonstrates that a noninertial policy rule with serially correlated errors can be nearly observationally equivalent to an inertial policy rule with serially uncorrelated errors. Orphanides (2001) demonstrates analytically how a misspecification of the Fed's policy rule can lead to the appearance of a larger inertia parameter.

3. Simulation Procedure

The parameter values used in the quantitative simulations are described in Box 3. I consider two possibilities for the exogenous time series that defines true potential output in

11. Qualitatively similar results would be obtained with other regression algorithms that assign more weight to recent data, such as moving-window least squares or discounted least squares. For details, see Lansing (2000).

12. For details, see Baxter and King (1999) and Christiano and Fitzgerald (1999).

13. For quantitative demonstrations of this property, see de Brouwer (1998), Orphanides and van Norden (1999), and Christiano and Fitzgerald (1999).

14. Goodfriend (1985) shows how this econometric problem can lead to a spurious finding of partial adjustment in estimated money demand equations when the variables that govern demand (interest rates and transactions) are subject to serially correlated measurement error.

Box 2
REDUCED-FORM VERSION OF THE MODEL

Following the procedure outlined in Lansing (2000), the reduced-form version of the aggregate demand equation can be written as follows:

$$(7)\qquad y_t - \overline{y}_t = \left[\frac{a_1 + a_r\left[(1-\rho)\,g_y/2 - 2\gamma\right]}{1+\gamma a_r}\;\; \frac{a_2 - a_r\,(1-\rho)\,g_y\phi/2}{1+\gamma a_r}\;\; \frac{a_r\left[(1-\rho)\,g_\pi/2 - 1\right]}{1+\gamma a_r}\;\; \frac{a_r\,(1+\rho)\,/2}{1+\gamma a_r}\right] \begin{bmatrix} y_{t-1} - \overline{y}_{t-1} \\ y_{t-2} - \overline{y}_{t-2} \\ \pi_{t-1} - \overline{\pi} \\ i_{t-1} - (\overline{r} + \overline{\pi}) \end{bmatrix}$$

$$+ \left[1 \;\; \frac{a_r\,(1-\rho)\,g_y/2}{1+\gamma a_r} \;\; \frac{-a_r\,(1-\rho)\,g_y\phi/2}{1+\gamma a_r}\right] \begin{bmatrix} v_t \\ \overline{y}_{t-1} - \overline{x}_{t-1} \\ \overline{y}_{t-2} - \overline{x}_{t-2} \end{bmatrix},$$

where $\overline{y}_{t-1} - \overline{x}_{t-1}$ represents the Fed's real-time error in measuring potential output in period $t-1$. The last two terms in equation (7) show how the real-time errors are transmitted to the true output gap $y_t - \overline{y}_t$.

The reduced-form Phillips curve is given by:

$$(8)\qquad \pi_t = \pi_{t-1} + 2\gamma\left(y_{t-1} - \overline{y}_{t-1}\right) + z_t\,,$$

which shows that the Fed's real-time measurement errors affect inflation only indirectly through their influence on the true output gap $y_t - \overline{y}_t$.

Combining equations (5) and (6), we can rewrite the Fed's policy rule as

$$(9)\qquad i_t = \left[\,\rho \quad (1-\rho)\,(\overline{r}+\overline{\pi}) \quad (1-\rho)\,g_\pi \quad (1-\rho)\,g_y \quad -(1-\rho)\,g_y\phi\,\right] \begin{bmatrix} i_{t-1} \\ 1 \\ \pi_{t-1} - \overline{\pi} \\ y_{t-1} - \overline{y}_{t-1} \\ y_{t-2} - \overline{y}_{t-2} \end{bmatrix}$$

$$+ \left[\,(1-\rho)\,g_y \quad -(1-\rho)\,g_y\phi\,\right] \begin{bmatrix} \overline{y}_{t-1} - \overline{x}_{t-1} \\ \overline{y}_{t-2} - \overline{x}_{t-2} \end{bmatrix},$$

where the last two terms show how the Fed's real-time measurement errors influence the setting of the current funds rate i_t. An econometrician who uses final data to estimate the Fed's policy rule is implicitly imposing the restriction $\overline{x}_t = \overline{y}_t$ for all *t*. This restriction causes the last two terms in equation (9) to drop out, thereby creating a missing variable problem.

The reduced-form version of the model is defined by equations (7), (8), and (9), together with the regression algorithm that defines the Fed's real-time potential output series $\{\overline{x}_t\}$ from observations of $\{y_s\}_{s=t_0}^{s=t}$.

Box 3
PARAMETER VALUES FOR QUANTITATIVE SIMULATIONS

Structural Parameters[a]					Standard Deviation of Shocks[b]		Policy Rule Parameters[c]				
a_1	a_2	a_r	γ	$\overline{r}$	σ_v	σ_z	ρ	g_π	g_y	ϕ	$\overline{\pi}$
1.25	–0.35	–0.2	0.04	0.03	0.0045	0.0050	0[d]	1.5	1.0	0	0.043[e]

[a] Values are taken from Lansing (2000), who estimates these parameters using quarterly U.S. data for the period 1966:Q1 to 2001:Q2.
[b] Standard deviations of the two shocks are chosen such that the standard deviations of the output gap and inflation in the simulations are close to the corresponding values in U.S. data over the period 1966:Q1 to 2001:Q2.
[c] Values are taken from Lansing (2000) and approximate a modified version of the original Taylor (1993) rule. The modified version (analyzed by Taylor (1999)) involves a stronger response to the output gap.
[d] Value indicates no interest rate smoothing by Fed policymakers in the model.
[e] Value matches the sample mean of U.S. inflation from 1966:Q1 to 2001:Q2. The annualized inflation rate is measured by $4\ln\left(P_t/P_{t-1}\right)$, where P_t is the GDP price index in quarter *t*.

the model. The first, shown in Figure 1A, is a segmented linear trend fitted to U.S. real GDP data of vintage 2001:Q3.[15] The sample starts at $t_0 = 1947{:}Q1$. I allow for two structural breaks at $t_1 = 1973{:}Q4$ and $t_2 = 1995{:}Q4$. The first breakpoint is consistent with research on the dating of the 1970s productivity slowdown. The dating of the second breakpoint is consistent with the analyses of Oliner and Sichel (2000) and Gordon (2000) and is intended to capture the start of the so-called "new economy." In Figure 1A, the postulated new economy break involves a slope change only; there is no attendant shift in the level of potential output. An unrestricted linear regression over the period 1996:Q1 to 2001:Q2 would imply a downward shift in the level of potential output at 1995:Q4. This outcome seems inconsistent with the mainstream new economy view that I am trying to capture here.[16] The second possibility for the true potential output series, shown in Figure 1B, is a simple linear trend with no breakpoints fitted over the entire sample period, 1947:Q1 to 2001:Q2. This alternative series allows me to gauge the impact of sudden trend shifts on the estimated value of the inertia parameter ρ in the model simulations.

For each of the two potential output series, I simulate the model 1,000 times with shock realizations drawn randomly from independent normal distributions with the standard deviations shown in Box 3. Each simulation starts from the steady state at $t_0 = 1947{:}Q1$ and runs for 218 periods (the number of quarters from 1947:Q1 to 2001:Q2). Fuhrer and Moore (1995) argue that the federal funds rate can be viewed as the primary instrument of monetary policy only since the mid-1960s. Before then, the funds rate traded below the Federal Reserve discount rate. Based on this reasoning, the Fed's algorithm for constructing the real-time potential output series is placed in service at 1966:Q1. Prior to this date, I set the real-time measure of potential output equal to true potential output. Thus I assume that the U.S. economy was fluctuating around its steady state before the Fed sought to exert control through the federal funds rate in the mid-1960s. Occasionally, a particular sequence of shock realizations will cause the federal funds rate to become negative. Overall, however, I find that this occurs in only about 3 percent of the periods during the simulations.

Each model simulation produces a set of artificial data on interest rates, inflation, and real output. Given the artificial data, I take the perspective of an econometrician who estimates the Fed's policy rule for two different sample periods. The first sample period runs from 1966:Q1 to 1979:Q2. The second sample period runs from 1980:Q1 to 2001:Q2. These sample periods are representative of those typically used in the empirical policy rule literature.[17] I consider two possible misspecifications of the econometrician's regression equation. First, he uses a final-data potential output series in place of the Fed's real-time potential output series. I assume that the final-data potential output series coincides exactly with the true potential output series.[18] Second, the econometrician may adopt a functional form that differs from the Taylor-type rule that is being used by Fed policymakers.

4. Results: The Illusion of Interest Rate Smoothing

The results of the quantitative simulations are summarized in Tables 1 through 4 and Figures 2 through 7.

Table 1 presents the results of policy rule regressions on model-generated data for the case where the econometrician employs the correct functional form for the regression equation, i.e., a Taylor-type rule. In this case, the only misspecification involves the use of a final-data potential output series in place of the real-time series. The table shows that the estimated inertia parameter $\hat{\rho}$ is positive and statistically significant in nearly all of the 1,000 trials, even though the Fed is actually using a Taylor-type rule with $\rho = 0$.[19] The average magnitude of the spurious regression coefficient is around 0.3 in the first sample period and 0.4 in the second sample period. As noted in Section 2.2, the econometrician's estimate of the inertia parameter is biased upwards because the lagged funds rate helps compensate for missing variables that influence the period-by-period setting of the funds rate. The missing variables are the Fed's serially correlated real-time measurement errors. With the inclusion of the lagged funds rate, the empirical fit of the misspecified rule is actually quite good; the average $\bar{R}^2$ statistic exceeds 90 percent.[20]

15. The data are described in Croushore and Stark (1999).

16. Allowing for a downward shift in the level of potential output at 1995:Q4 has a negligible impact on the quantitative results.

17. Empirical policy rule studies typically allow for a break in the monetary policy regime sometime in the late 1970s. For evidence of such a break, see Estrella and Fuhrer (1999).

18. Qualitatively similar results are obtained if the econometrician constructs his own final-data potential output series either by applying the HP filter over the entire sample period 1947:Q1 to 2001:Q2 or by fitting a quadratic trend over the same period.

19. If the t-statistic associated with a given regression coefficient is above the critical value of 1.96, then the econometrician rejects the null hypothesis that the true value of that coefficient is zero.

20. The $\bar{R}^2$ statistic gauges the fraction of the variance in the federal funds rate that can be explained by the variables on the right-hand side of the regression equation (with a correction factor applied for the number of regressors).

Figure 1
U.S. Real GDP, 1947:Q1 to 2001:Q2

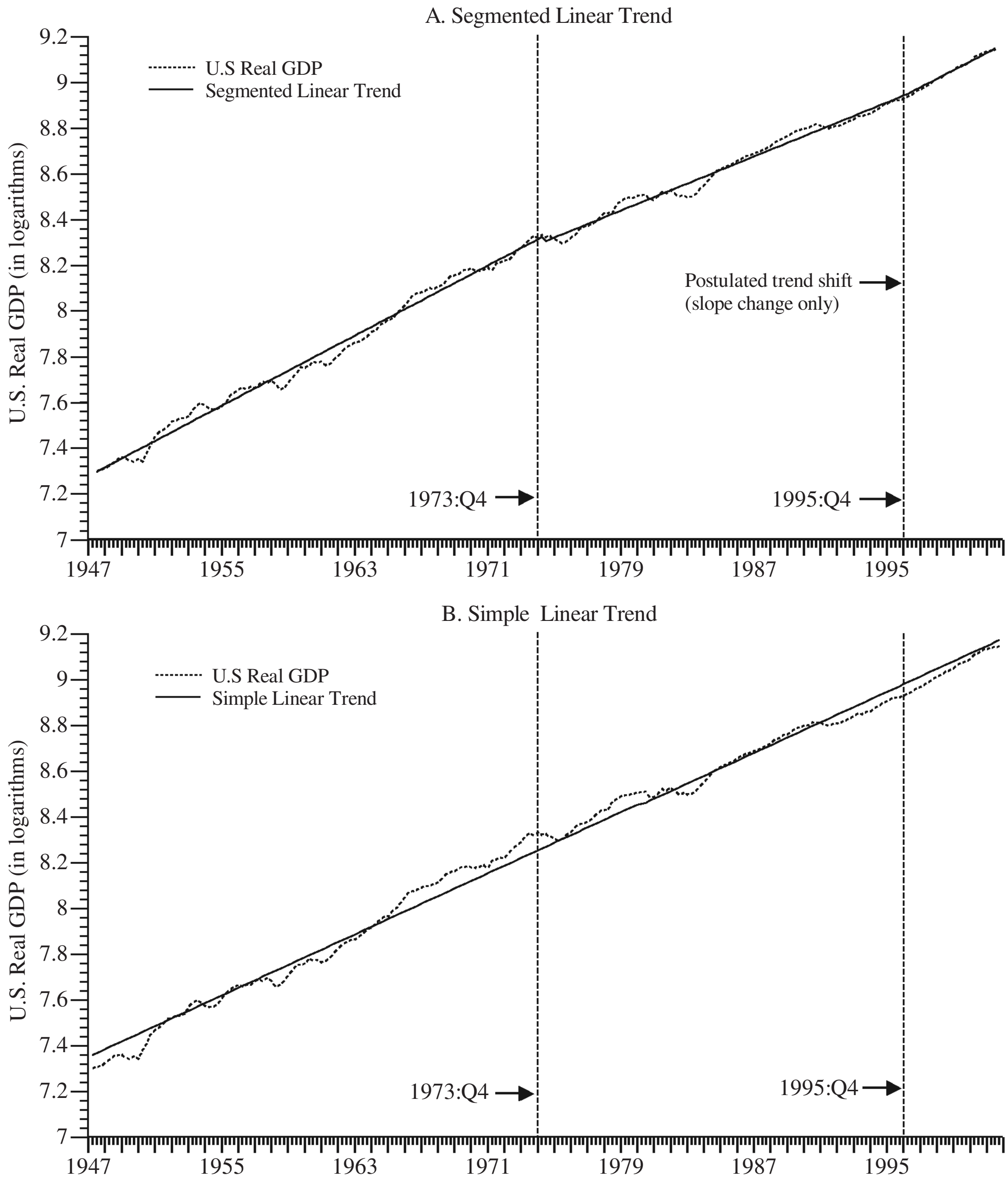


Table 1 also shows that the average point estimate of the inertia parameter does not change much when the model is simulated without trend shifts. This is due to the nature of the regression algorithm (the HP filter) that is used to construct the Fed's real-time estimate of potential output. As discussed further below, the regression algorithm gives rise to serially correlated real-time measurement errors even when there is no fundamental change in the underlying economy.

The average point estimate for the inflation response coefficient $\hat{g}_\pi$ in Table 1 is around 1.4, only slightly below the true value of $g_\pi = 1.5$. The estimated coefficient is statistically significant in 100 percent of the trials. Hence, the econometrician's use of the final-data potential output series does not lead to significantly incorrect conclusions about the Fed's desired response to inflation during either of the two sample periods. This point relates to the studies by Perez (2001) and Mehra (2001), each of which investigates whether the Fed's desired response to inflation was less aggressive during the 1970s. Both authors note that policy rule regressions based on final data suggest that the desired funds rate moved less than one-for-one with

TABLE 1
POLICY RULE REGRESSIONS ON MODEL-GENERATED DATA

Actual Taylor-type rule:
$i_t = 0 i_{t-1} + (1-0)\,[0.0085 + 1.5\,\pi_{t-1} + 1.0\,(y_{t-1} - \overline{x}_{t-1})]$

Estimated Taylor-type rule:
$i_t = \hat{\rho} i_{t-1} + (1-\hat{\rho})\left[\hat{g}_0 + \hat{g}_\pi\,\pi_{t-1} + \hat{g}_y\left(y_{t-1} - \overline{y}_{t-1}\right)\right] + \varepsilon_t$

	Model with Trend Shifts				Model without Trend Shifts			
Model Sample Period	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$
1966:Q1 to 1979:Q2								
Average point estimate	0.34	0.011	1.40	0.22	0.29	0.014	1.37	0.22
Standard deviation of point estimate	0.12	0.008	0.16	0.22	0.12	0.007	0.14	0.20
Average *t*-statistic	3.81	2.98	18.0	1.99	3.53	4.69	21.5	2.39
% trials with $t > 1.96$	91.1%	66.3%	100%	46.3%	85.2%	82.5%	100%	31.1%
	Average $\bar{R}^2 = 0.92$, Average $\sigma_\varepsilon = 0.006$				Average $\bar{R}^2 = 0.94$, Average $\sigma_\varepsilon = 0.005$			
1980:Q1 to 2001:Q2								
Average point estimate	0.39	0.015	1.37	0.03	0.39	0.014	1.37	0.04
Standard deviation of point estimate	0.09	0.004	0.09	0.13	0.09	0.004	0.09	0.14
Average *t*-statistic	5.75	6.36	28.5	0.08	5.65	6.06	28.4	0.24
% trials with $t > 1.96$	99.5%	96.2%	100%	31.5%	99.2%	94.8%	100%	31.1%
	Average $\bar{R}^2 = 0.95$, Average $\sigma_\varepsilon = 0.006$				Average $\bar{R}^2 = 0.95$, Average $\sigma_\varepsilon = 0.006$			

Notes: Model statistics are based on 1,000 simulations. σ_ε is the standard deviation of a serially uncorrelated zero-mean error ε_t, added for the purpose of estimation. $\overline{x}_t$ = Fed's real-time potential output defined by the HP filter with λ = 1,600. $\overline{y}_t$ = econometrician's final-data potential output defined by a segmented linear trend (Figure 1A) or a simple linear trend (Figure 1B).

inflation (or expected inflation) during the 1970s.[21] Perez (2001) shows that a policy rule estimated using reconstructed historical data yields the opposite conclusion, i.e., the desired funds rate moved *more* than one-for-one with expected inflation during the 1970s.[22] Perez adopts a rule specification where the Fed reacts to real-time forecasts of *future* inflation. The real-time forecasts appear to have systematically underpredicted actual inflation during the 1970s. In contrast, the model-based regressions performed here apply to an economy where the Fed reacts to *lagged* inflation which I assume is observed without error. Also using reconstructed historical data, Mehra (2001) finds that the desired response to inflation during the 1970s was less than one-for-one for a rule where the Fed reacts to the quarterly inflation rate, but not significantly different from one-for-one for a rule where the Fed reacts to a "smoothed" inflation rate defined by a four-quarter moving average of the quarterly inflation rate. Both of these studies demonstrate the general point, also emphasized here, that empirical estimates of the Fed's policy rule are sensitive to the data vintage and the functional form adopted by the econometrician.

The average point estimate for the output gap response coefficient $\hat{g}_y$ in Table 1 is substantially below the true value of $g_y = 1.0$, particularly during the second sample period. Moreover, the estimated coefficient is statistically significant in less than one-half of the trials. This result shows that quarterly variations in the final-data output gap do not hold much explanatory power for quarterly movements in the model funds rate.

As a benchmark for comparison, Table 2 presents the results of regressing a Taylor-type policy rule on "final" U.S. data of vintage 2001:Q3. For both sample periods, the estimated coefficient $\hat{\rho}$ on the lagged federal funds rate is around 0.8 and strongly significant. This confirms the statement made earlier that regressions based on final data imply a high degree of policy inertia at quarterly frequency. The table also shows that the estimated values of the other policy rule coefficients, $\hat{g}_0$, $\hat{g}_\pi$, and $\hat{g}_y$, differ substantially across the two sample periods.[23] Placing an

21. For discussions of this result, see Taylor (1999) and Clarida, et al. (2000).

22. Perez (2001) obtains this result for two different sample periods: the first runs from 1975:Q1 to 1979:Q2 and the second runs from 1969:Q1 to 1976:Q3.

23. The regression coefficient $\hat{g}_0$ is an estimate of the combined coefficient $g_0 \equiv \overline{r} + \overline{\pi}\,(1 - g_\pi)$. Without additional information, the data cannot separately identify the values of $\overline{r}$ and $\overline{\pi}$.

TABLE 2
POLICY RULE REGRESSIONS ON FINAL U.S. DATA (VINTAGE 2001:Q3)

Estimated Taylor-type rule:
$i_t = \hat{\rho} i_{t-1} + (1-\hat{\rho})\left[\hat{g}_0 + \hat{g}_\pi \pi_{t-1} + \hat{g}_y (y_{t-1} - \overline{y}_{t-1})\right] + \varepsilon_t$

	Regression with Trend Shifts				Regression without Trend Shifts			
U.S. Sample Period	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$
1966:Q1 to 1979:Q2								
U.S. point estimate	0.80	0.033	0.45	1.19	0.70	–0.026	0.66	1.06
U.S. *t*-statistic	8.82	1.69	1.37	2.45	7.38	–1.36	3.26	3.81
	$\bar{R}^2 = 0.83,\ \sigma_\varepsilon = 0.009$				$\bar{R}^2 = 0.84,\ \sigma_\varepsilon = 0.009$			
1980:Q1 to 2001:Q2								
U.S point estimate	0.77	0.025	1.38	0.24	0.74	0.037	1.17	0.32
U.S. *t*-statistic	13.0	2.70	6.02	0.98	12.0	3.43	4.92	1.66
	$\bar{R}^2 = 0.91,\ \sigma_\varepsilon = 0.010$				$\bar{R}^2 = 0.91,\ \sigma_\varepsilon = 0.010$			

Notes: σ_ε is the standard deviation of a serially uncorrelated zero-mean error ε_t added for the purpose of estimation. $\overline{y}_t$ = final-data potential output defined by a segmented linear trend (Figure 1A) or a simple linear trend (Figure 1B).

economic interpretation on these results is problematic, however, because policymakers did not see the final data—instead they saw the data that was available at the time policy decisions were made. The real-time data may have presented a very different picture of the economy. Indeed, recent studies by Croushore and Stark (1999), Orphanides (2000, 2001), Perez (2001), and Mehra (2001) make it clear that retrospective analyses based on real-time data often can lead to conclusions that differ from those based on final data.

Figure 2 compares the average trajectory of the Fed's real-time potential output series $\{\overline{x}_t\}$ with the true potential output series $\{\overline{y}_t\}$ for the case where the model includes trend shifts. In the periods after the first trend shift at 1973:Q4, the incoming data on real output y_t (which are fully observable to policymakers) start to plot below the Fed's previously estimated trend because of the unobserved structural break. Fed policymakers interpret the data as evidence of a recession. Following the advice of their policy rule, they lower the federal funds rate in response to the perceived negative output gap. The drop in the funds rate stimulates aggregate demand. Stronger demand, combined with the abrupt reduction in the economy's productive capacity, causes the true output gap to become positive (Figure 3). Later, as more data are received, the Fed adjusts its estimated trend, shrinking the size of the perceived negative output gap.[24]

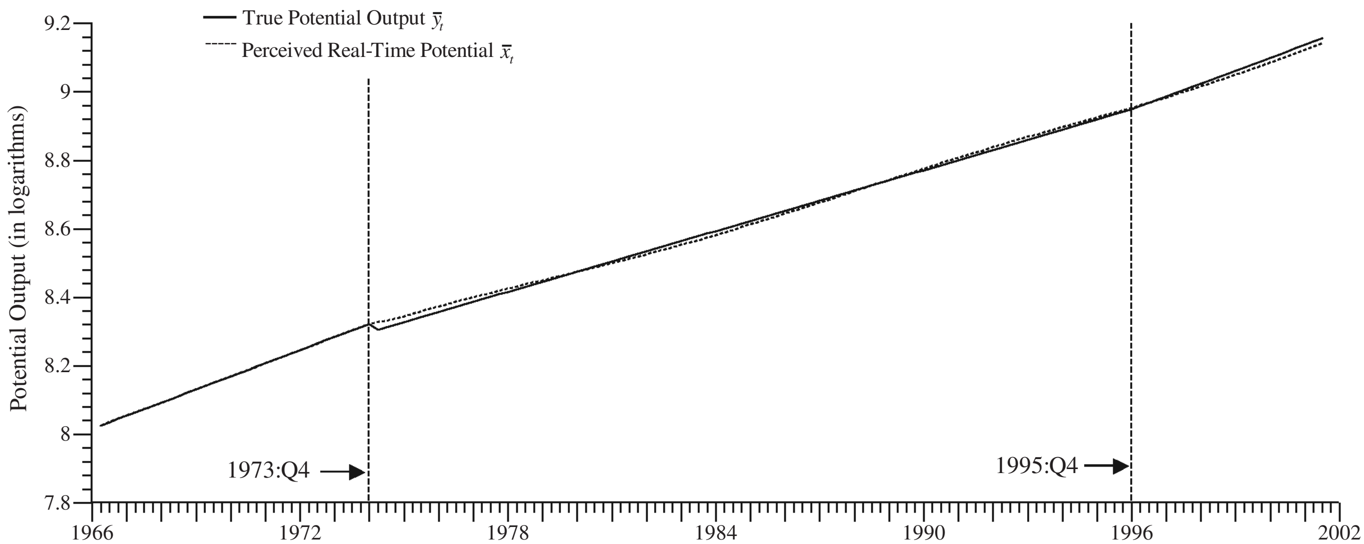
By the time of the second trend shift at 1995:Q4, the divergence between the true gap and the perceived gap has been reduced but not eliminated (Figure 3). In the periods after the second trend shift, the incoming data on y_t start to plot above the Fed's previously estimated trend because of the unobserved structural break. Fed policymakers interpret the data as evidence of a boom. Following the advice of their policy rule, they raise the federal funds rate in an effort to restrain aggregate demand. This action, combined with expansion in the economy's productive capacity, pushes the true output gap into negative territory while the Fed's perceived gap becomes positive.

The divergence between the perceived real-time gap and the true gap shown in Figure 3 represents the Fed's real-time measurement error. The divergence narrows over time as the Fed's regression algorithm detects the trend shift. Figure 4 plots the trajectory of the real-time error. The real-time errors are highly serially correlated with an autocorrelation coefficient of 0.99. Negative errors tend to be followed by negative errors while positive errors tend to be followed by positive errors. The standard deviation of the real-time errors over the period 1966:Q1 to 2001:Q2 is 2.6 percent (averaged over 1,000 simulations). The statistical properties of the real-time errors are similar to those documented by Orphanides and van Norden (1999) for a variety of real-time methods of trend estimation. This result suggests that the basic nature of the results does not depend on the particular regression algorithm used by Fed policymakers in the model.[25]

24. Further details on the behavior of the output gap, inflation, and the federal funds rate during the model simulations can be found in Lansing (2000).

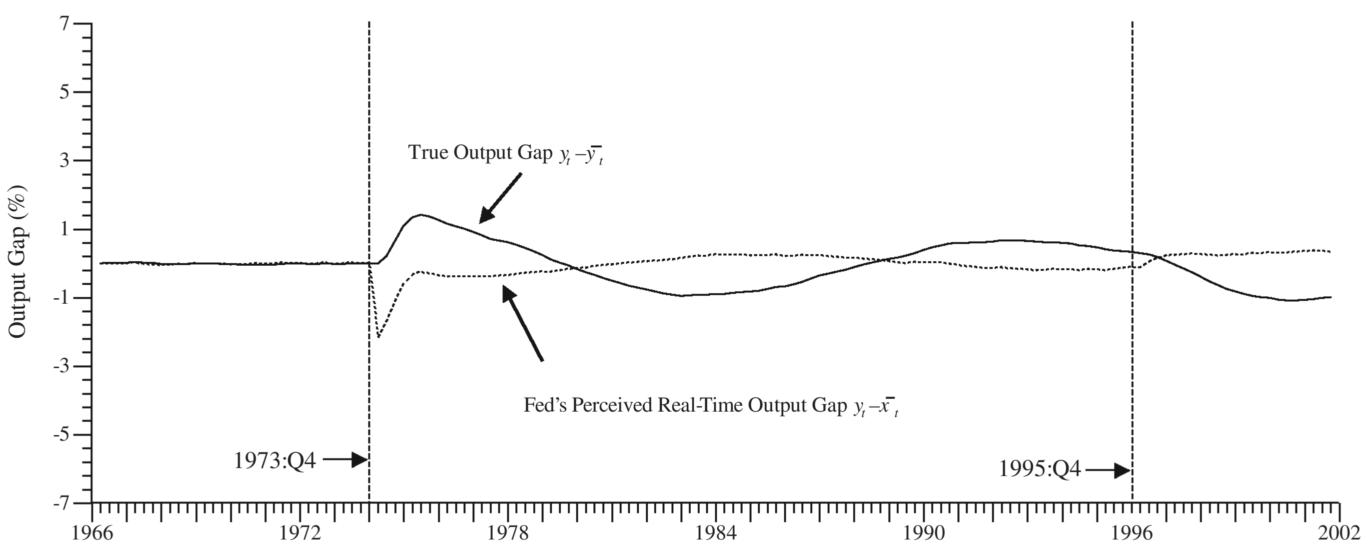
25. The standard deviation of the final-data output gap in the model is 2.37 percent. The standard deviation of the final-data output gap in U.S. data (defined by a segmented linear trend) is 2.24 percent. For additional details, see Lansing (2000).

Figure 2
Model Potential Output



Note: Average trajectory taken from 1,000 simulations, model with trend shifts.

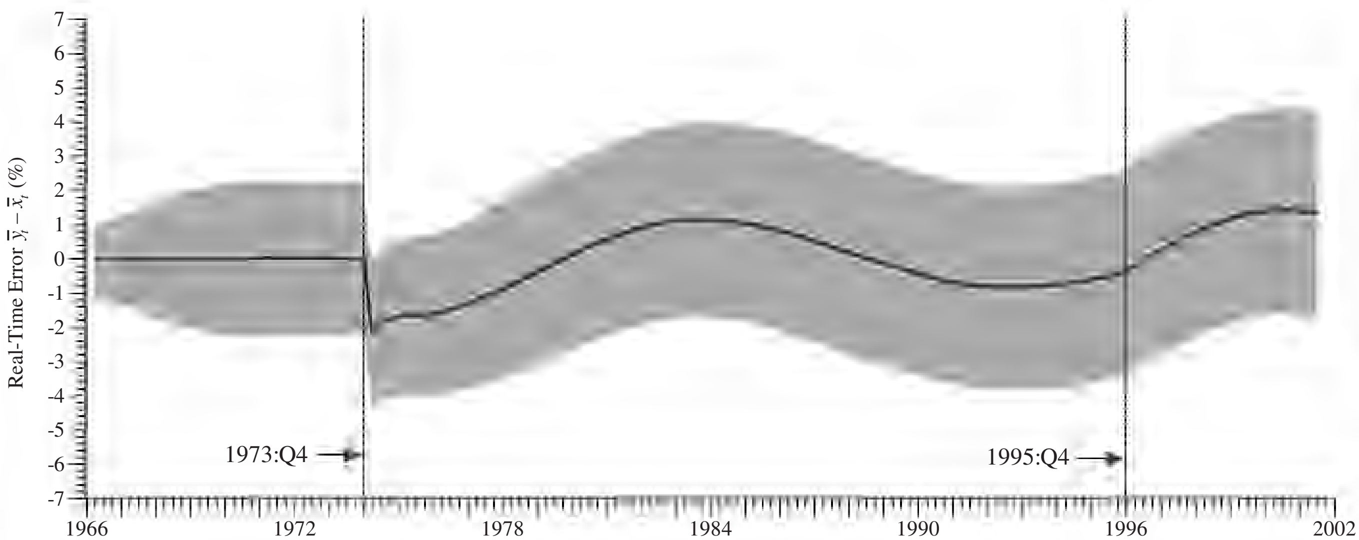
Figure 3
Model Output Gap



Note: Average trajectory taken from 1,000 simulations, model with trend shifts.

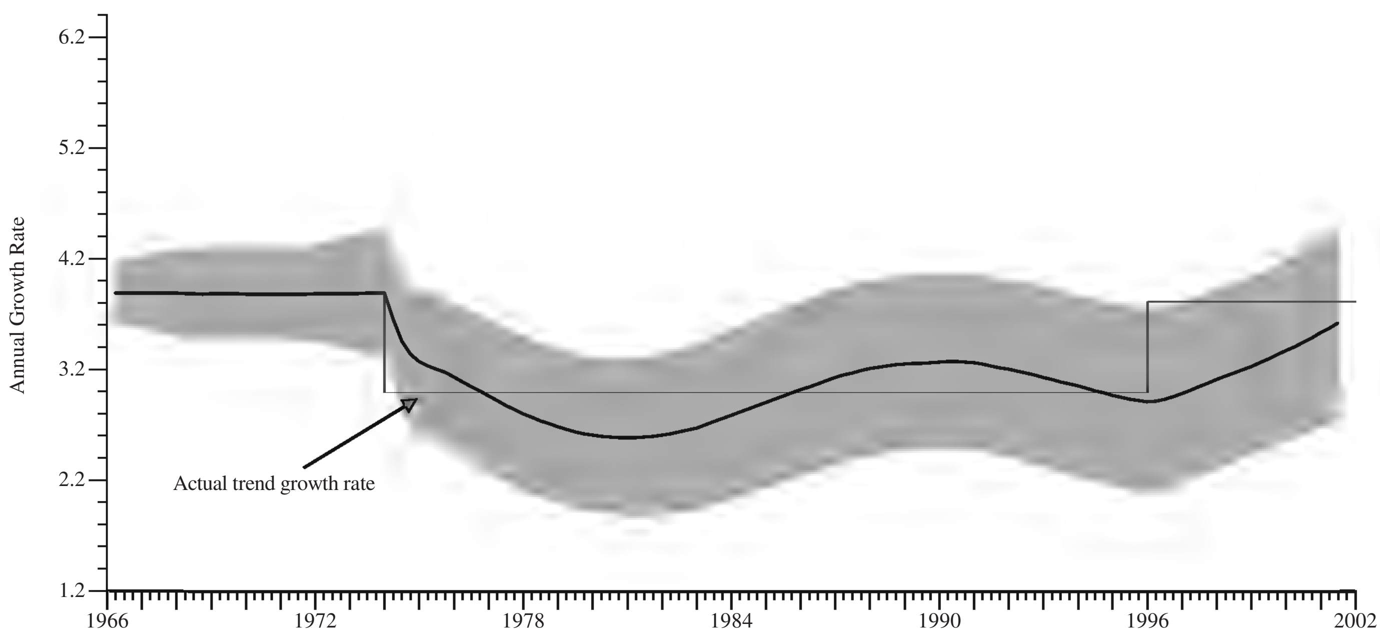
Figure 5 shows that the Fed's regression algorithm exhibits overshooting behavior. Overshooting occurs because the HP filter assigns a relatively high weight to the most recent data. If a sequence of recent data observations happens to fall mostly above or mostly below the Fed's previously estimated trend, then the Fed's real-time estimate of potential output can undergo a substantial revision even when there is no trend shift in the underlying economy. This point is illustrated in Figures 4 and 5 by the fairly wide standard error bands that can be observed around the average trajectories even before the first trend shift takes place at 1973:Q4. The errors induced by the Fed's regression algorithm during normal times are a tradeoff for being able to detect a trend shift more quickly when it does occur.

FIGURE 4
FED'S REAL-TIME MEASUREMENT ERROR



Note: Average trajectory taken from 1,000 simulations, model with trend shifts. Gray band shows ±1 standard deviation.

FIGURE 5
FED'S ESTIMATED GROWTH RATE OF POTENTIAL OUTPUT



Note: Average trajectory taken from 1,000 simulations, model with trend shifts. Gray band shows ±1 standard deviation.

TABLE 3
POLICY RULE REGRESSIONS ON MODEL-GENERATED DATA

Actual Taylor-type rule:
$i_t = 0 i_{t-1} + (1-0)\{0.0085 + 1.5\pi_{t-1} + 1.0[(y_{t-1} - \overline{x}_{t-1}) - 0(y_{t-2} - \overline{x}_{t-2})]\}$

Estimated general rule:
$i_t = \hat{\rho} i_{t-1} + (1-\hat{\rho})\left\{\hat{g}_0 + \hat{g}_\pi \pi_{t-1} + \hat{g}_y\left[(y_{t-1} - \overline{y}_{t-1}) - \hat{\phi}(y_{t-2} - \overline{y}_{t-2})\right]\right\} + \varepsilon_t$

	Model with Trend Shifts					Model without Trend Shifts				
Model Sample Period	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\phi}$	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\phi}$
1966:Q1 to 1979:Q2										
Average point estimate	0.39	0.006	1.51	1.39	0.88	0.33	0.010	1.47	1.31	0.88
Standard deviation of point estimate	0.11	0.008	0.16	0.44	0.16	0.10	0.006	0.12	0.31	0.13
Average *t*-statistic	5.26	1.88	19.1	4.26	14.5	5.36	3.88	25.9	5.74	18.5
% trials with $t > 1.96$	98.2%	45.4%	100%	99.6%	99.7%	98.1%	75.7%	100%	100%	100%
	Average $\bar{R}^2 = 0.95$, Average $\sigma_\varepsilon = 0.005$					Average $\bar{R}^2 = 0.96$, Average $\sigma_\varepsilon = 0.004$				
1980:Q1 to 2001:Q2										
Average point estimate	0.40	0.010	1.49	1.40	1.00	0.40	0.009	1.48	1.40	0.99
Standard deviation of point estimate	0.07	0.004	0.08	0.29	0.08	0.07	0.003	0.08	0.28	0.08
Average *t*-statistic	8.04	5.02	34.3	6.39	31.6	7.97	4.74	34.7	6.56	31.7
% trials with $t > 1.96$	100%	90.1%	100%	100%	100%	100%	87.7%	100%	100%	100%
	Average $\bar{R}^2 = 0.97$, Average $\sigma_\varepsilon = 0.004$					Average $\bar{R}^2 = 0.98$, Average $\sigma_\varepsilon = 0.004$				

Notes: Model statistics are based on 1,000 simulations. σ_ε is the standard deviation of a serially uncorrelated zero-mean error ε_t added for the purpose of estimation. $\overline{x}_t =$ Fed's real-time potential output defined by the HP filter with $\lambda = 1{,}600$. $\overline{y}_t =$ econometrician's final-data potential output defined by a segmented linear trend (Figure 1A) or a simple linear trend (Figure 1B).

Table 3 presents the results of policy rule regressions on model-generated data for the case where the econometrician adopts the wrong functional form for the Fed's policy rule. The econometrician estimates a general rule that includes the twice-lagged output gap $(y_{t-2} - \overline{y}_{t-2})$. As before, the econometrician employs a final-data potential output series in place of the Fed's real-time series. The results are broadly similar to those reported in Table 1. Notice, however, that the average magnitude of the spurious regression coefficient $\hat{\rho}$ is slightly higher than before. As would be expected, the econometrician's use of the wrong functional form contributes to the upward bias in $\hat{\rho}$. This effect is partially offset, however, by the presence of the twice-lagged output gap, which helps to reduce the dependence on the lagged funds rate when fitting the misspecified rule to the data. The twice-lagged gap is strongly significant in nearly all of the trials with an average point estimate of $\hat{\phi} \approx 1$. The intuition for the spurious significance of the twice-lagged gap is straightforward. Since the true output gap is highly serially correlated, a point estimate of $\hat{\phi} \approx 1$ allows successive true output gaps (which contain little explanatory power for i_t) to offset one another. Given that the average point estimates imply $\hat{g}_\pi \approx \hat{g}_y$ and $\hat{\phi} \approx 1$, the econometrician may conclude that Fed policymakers are using a smoothed nominal income growth rule when, in fact, they are using an unsmoothed Taylor-type rule.[26]

Table 4 presents the results of regressing the general policy rule on final U.S. data (vintage 2001:Q3). The estimated coefficient $\hat{\rho}$ on the lagged funds rate is again in the neighborhood of 0.8. The estimated coefficient $\hat{\phi}$ on the twice-lagged output gap is statistically significant in both sample periods. In the sample period that runs from 1980:Q1 to 2001:Q2, it is unlikely that one could reject the hypothesis of $\hat{g}_\pi = \hat{g}_y$ and $\hat{\phi} = 1$. Hence, just as in the model-based regressions described above, the time path of the U.S. federal funds rate since 1980 appears to be well approximated by a smoothed nominal income growth rule.[27] Unlike the model, however, we cannot know for sure what policy rule (if any) was being used by Fed policymakers during this sample period.

It is important to recognize that the value of $\hat{\rho}$ reported in Tables 1 and 3 is an *average* point estimate computed over the course of many simulations. In any given simula-

26. Recall that a nominal income growth rule can be represented as a special case of equation (5) with $g_\pi = g_y$ and $\phi = 1$. See the appendix for details.

27. This result confirms the findings of McCallum and Nelson (1999).

TABLE 4
POLICY RULE REGRESSIONS ON FINAL U.S. DATA (VINTAGE 2001:Q3)

Estimated general rule:
$$i_t = \hat{\rho} i_{t-1} + (1-\hat{\rho})\left\{\hat{g}_0 + \hat{g}_\pi \pi_{t-1} + \hat{g}_y\left[\left(y_{t-1} - \overline{y}_{t-1}\right) - \hat{\phi}\left(y_{t-2} - \overline{y}_{t-2}\right)\right]\right\} + \varepsilon_t$$

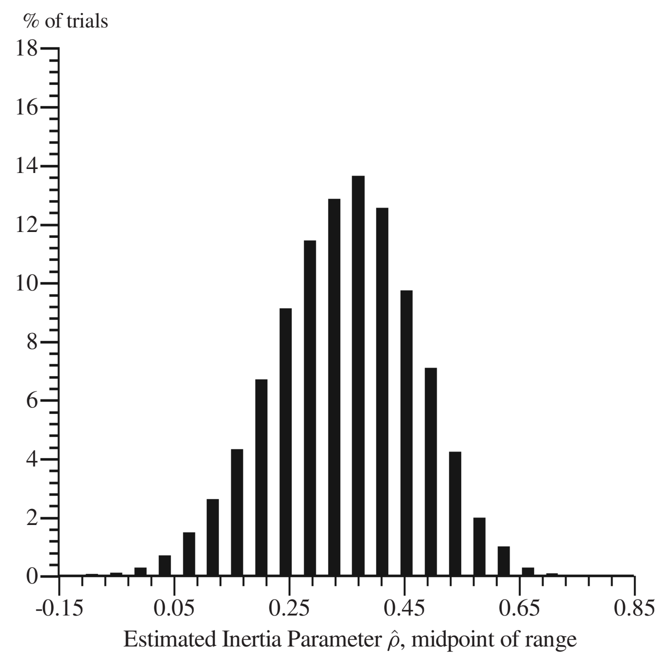
	Regression with Trend Shifts					Regression without Trend Shifts				
U.S. Sample Period	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\phi}$	$\hat{\rho}$	$\hat{g}_0$	$\hat{g}_\pi$	$\hat{g}_y$	$\hat{\phi}$
1966:Q1 to 1979:Q2										
U.S. point estimate	0.93	0.033	0.40	8.50	0.73	0.80	–0.044	0.79	2.68	0.51
U.S. t-statistic	10.1	0.64	0.45	0.73	6.37	7.61	–1.27	2.60	1.64	3.06
	$\bar{R}^2 = 0.86,\ \sigma_\varepsilon = 0.008$					$\bar{R}^2 = 0.86,\ \sigma_\varepsilon = 0.009$				
1980:Q1 to 2001:Q2										
U.S point estimate	0.76	0.021	1.51	1.63	0.91	0.74	0.030	1.37	1.54	0.85
U.S. t-statistic	13.2	2.33	6.69	2.24	6.76	12.3	2.70	5.41	2.40	6.44
	$\bar{R}^2 = 0.91,\ \sigma_\varepsilon = 0.010$					$\bar{R}^2 = 0.92,\ \sigma_\varepsilon = 0.010$				

Notes: σ_ε is the standard deviation of a serially uncorrelated zero-mean error ε_t added for the purpose of estimation. $\overline{y}_t =$ final-data potential output defined by a segmented linear trend (Figure 1A) or a simple linear trend (Figure 1B).

tion, the estimated coefficient on the lagged funds rate may turn out to be higher or lower than the average value. Figures 6 and 7 show the distribution of point estimates generated by the model for each of the two sample periods.[28] The mean of the distribution is slightly higher in the second sample period because the Fed's regression algorithm has been running longer at that point.[29] This increases the probability that the regression algorithm will generate serially correlated real-time measurement errors. For the sample period that runs from 1966:Q1 to 1979:Q2 (Figure 6), the 95 percent confidence interval for the estimated inertia parameter ranges from a low of 0.09 to a high of 0.57.[30] For the sample period that runs from 1980:Q1 to 2001:Q2 (Figure 7), the 95 percent confidence interval ranges from a low of 0.20 to a high of 0.57. These confidence intervals suggest that the model-generated distributions can be approximated by standard normal distributions with the means and standard deviations shown in Tables 1 and 3.

In contrast to the model simulations, the point estimate for the U.S. inertia parameter reported in Tables 2 and 4

FIGURE 6
DISTRIBUTION OF POINT ESTIMATES GENERATED BY MODEL WITH TREND SHIFTS, SIMULATED SAMPLE PERIOD 1966:Q1 TO 1979:Q2

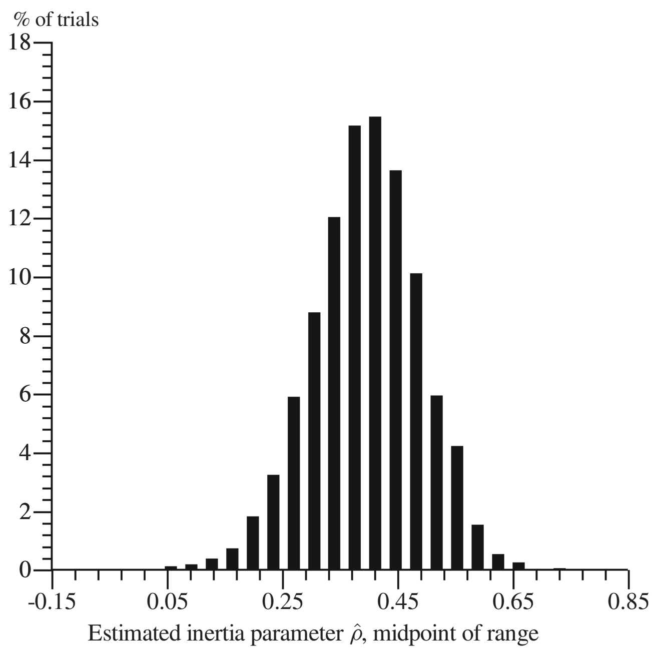


28. In constructing the histograms in Figures 6 and 7, the number of model simulations was increased to 5,000 in order to provide a more accurate picture of the true distribution governing the point estimates.

29. Recall that the Fed's regression algorithm is placed in service at 1966:Q1.

30. In other words, the estimated inertia parameter fell within this interval in 4,750 simulations out of a total of 5,000 simulations $\left(\frac{4,750}{5,000} = 0.95\right)$.

FIGURE 7
DISTRIBUTION OF POINT ESTIMATES GENERATED BY MODEL WITH TREND SHIFTS, SIMULATED SAMPLE PERIOD 1980:Q1 TO 2001:Q2



represents the outcome of a *single* regression. The point estimate is influenced by the particular sequence of random shocks that hit the U.S. economy during a given period of history. In Table 2, for example, the point estimate for the U.S. inertia parameter is $\hat{\rho} = 0.77$ when the sample period runs from 1980:Q1 to 2001:Q2 and the regression allows for trend shifts.

By comparing the U.S. point estimate to the distribution of point estimates generated by the model, one may conclude that there is less than a 1 percent chance that the model would produce a point estimate as high as $\hat{\rho} = 0.77$ during a single simulation. This tells us that the model cannot explain all of the inertia that we observe in the U.S. federal funds rate. Nevertheless, there is a 50 percent chance that the model would produce a point estimate as high as $\hat{\rho} = 0.39$ during a single simulation and a 25 percent chance that the model would produce a point estimate as high as $\hat{\rho} = 0.46$. Hence, the model can easily explain about one-half of the inertia that we observe in the U.S. federal funds rate.

One might argue that it makes sense for the model not to explain all of the U.S. inertia because the model abstracts from real-time errors in observing inflation or real output. Noise or measurement error in these variables may have influenced the setting of the U.S. federal funds rate during a given sample period. Indeed, Orphanides (2000) presents evidence which suggests that the Fed's real-time measures of inflation (based on a GDP price index) and real output were both too low in the early 1970s. The model also abstracts from any persistent changes in the real interest rate term $\bar{r}$ which appears in the policy rule equation (5). Rudebusch (2002) notes that a variety of economic influences (e.g., credit crunches, financial crises) can be interpreted as involving a temporary but persistent shift in the real interest rate. A perceived shift in $\bar{r}$ would induce movements in the funds rate that cannot be explained by observable changes in inflation or the output gap. Finally, the model abstracts from the difficult issue of determining which particular price index policymakers actually use when deciding whether current inflation has deviated from the Fed's long-run target rate. Unlike the model, there are many possible ways to define inflation in the U.S. economy.[31] The above considerations, if incorporated into the model, would contribute to an upward bias in the estimated inertia parameter beyond that which is due solely to the Fed's real-time errors in estimating potential output.

5. Conclusion

Empirical estimates of the Fed's policy rule based on quarterly U.S. data typically find that the lagged federal funds rate is a significant explanatory variable. The standard interpretation of this result is that the Fed intentionally "smoothes" interest rates, i.e., policymakers move gradually over time to bring the current level of the funds rate in line with a desired level that is determined by economic fundamentals. This paper employed simulations from a small macroeconomic model to demonstrate that efforts to identify the Fed's policy rule using regressions based on final data can create the illusion of interest rate smoothing behavior when, in fact, none exists. I showed that failure to account properly for policymakers' real-time perceptions about potential output can explain as much as one-half of the apparent degree of inertia in the U.S. federal funds rate. Interestingly, the simulated policy rule regressions suggested that Fed policymakers were using a smoothed nominal income growth rule when actually they were using an unsmoothed Taylor-type rule. Overall, the findings presented here lend support to a growing view within the economics profession that empirical results derived solely from an analysis of final data can provide a distorted picture of the monetary policy process.

31. This point has been emphasized recently by Federal Reserve Chairman Alan Greenspan (2001).

Appendix

Here I show that a nominal income growth rule can be represented by a special case of equation (5). Imposing $g_\pi = g_y = \theta > 0$, $\phi = 1$, and then rearranging yields

$$i_t^* = \overline{r} + \overline{\pi} + \theta \left[\pi_{t-1} + y_{t-1} - y_{t-2} - (\overline{x}_{t-1} - \overline{x}_{t-2}) - \overline{\pi} \right], \tag{A1}$$

where all rates are expressed initially on a quarterly basis. Quarterly inflation is given by $\pi_t = \ln(P_t / P_{t-1})$ for all t, where P_t is the GDP price index. We also have $y_t = \ln Y_t$ for all t, where Y_t is quarterly real GDP. Substituting these expressions into equation (A1) and rearranging yields

$$i_t^* = \overline{r} + \overline{\pi} + \theta \left[G_{t-1} - (\overline{x}_{t-1} - \overline{x}_{t-2}) - \overline{\pi} \right], \tag{A2}$$

where $G_{t-1} \equiv \ln(P_{t-1} Y_{t-1}) - \ln(P_{t-2} Y_{t-2})$ is the observed quarterly growth rate of nominal income.

Recall that $\overline{x}_{t-1}$ and $\overline{x}_{t-2}$ represent the Fed's estimate of the logarithm of potential output for the periods $t-1$ and $t-2$, respectively. Both of these quantities are computed at $t-1$, however, because the Fed runs a regression each period and updates its entire potential output series. Since $\overline{x}_{t-1}$ and $\overline{x}_{t-2}$ both lie on the best-fit trend line computed at $t-1$, we have $\hat{\mu}_{t-1} = \overline{x}_{t-1} - \overline{x}_{t-2}$, where $\hat{\mu}_{t-1}$ is the Fed's real-time estimate of the quarterly growth rate of potential output. Substituting this expression into equation (A2) yields

$$i_t^* = \overline{r} + \overline{\pi} + \theta \left[G_{t-1} - \left(\hat{\mu}_{t-1} + \overline{\pi} \right) \right], \tag{A3}$$

which shows that the desired federal funds rate i_t^* will be above its steady-state level $(\overline{r} + \overline{\pi})$ whenever observed nominal income growth G_{t-1} exceeds the target growth rate of $\hat{\mu}_{t-1} + \overline{\pi}$. Multiplying both sides of equation (A3) by 4 converts all quarterly rates to an annual basis.

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